Abstract

This paper describes the warehouse stock optimization using two optimization algorithms for products belonging to different classes according to ABC and XYZ analysis. For simulation mathematical tool Matlab was used. The basic system dynamics model of the warehouse was built according to system dynamics methodology and then validated. Several ordering strategies were analyzed with a goal of producing lower total warehousing costs than the actual costs provided by the observed company. Together with total costs two restrictions had to be considered: no stockouts should occur and the warehouse capacity should not be exceeded.

Keywords: system dynamics, simulation, model, optimization, warehouse

1. Introduction

Stocks are the stores of goods that an organization holds to meet a mismatch between supply and demand. A mismatch often occurs in an uncertain environment, where supply and demand are stochastic variables. If we recapitulate, stock acts as a buffer between supply and demand (Waters, 1997). Most organizations hold their stock in some kind of a warehouse.

Most production companies encounter the problem of how to optimize the warehouse processes. The most important issue with the optimization is a cost reduction of the warehousing processes to a minimum. Several different principles of the warehouse optimization were developed and are described in (Tompkins, Smith, 1998). Dealing with the problems of warehousing, we encounter several contradictory criteria. A warehouse, too large means greater amount of stock, bigger capital costs, more staff. Today the space itself is very valuable. A warehouse, too small can represent possible stockouts, it demands a reliable supplier etc.

Products stored in a warehouse also play an important role in a process of optimization of warehousing processes. They belong to different categories according to ABC and XYZ classification. ABC classification divides products into three categories according to their value
(Silver et al., 1998; Ljubić, 2000), while XYZ classification divides products into three categories according to the dynamics of their consumption (Ljubić, 2000). The dynamics of product consumption and the products value must be taken into consideration in order to improve the warehousing processes. We believe that there is a lack of optimization technology in use and that there are a number of possibilities of how to improve the warehousing processes. The warehouse personnel solve the complex problems mostly by using their experience.

This paper presents the simulation model, used to solve ordering strategy problems (when to place an order and how many products to order) in a medium-sized company in order to improve its warehousing processes. The model is based on the system dynamics methodology (Forrester, 1961), combined with a continuous simulation, and is a part of a decision support integrated system development. The purpose of the integrated system is to offer help to operative management in companies (Mulcahy, 1993). The main advantage of the presented model is in its increased man-machine interconnection, where a computer is capable of executing several simulation runs in a short period of time using different "what-if" scenarios. The planner or the warehouse personnel then choose the most appropriate scenario and modify it if needed, allowing the process of ordering to be faster and better.

A supply chain is generally viewed as a network of facilities that performs the procurement of raw material, its transformation to intermediate and end-products, distribution and selling of the end-products to end customers. (Petrovic et al., 1999). A warehouse with its ordering strategy is a crucial part of the supply chain. Role of the supply chain management is increasing with globalization, as modern technology allows the development of intra and inter-organizational networks. A case study of supply chain management group model building in the high-tech electronic industry is described in (Akkermans, 2001).

In the last two decades, computer simulation has become an indispensable tool for understanding the dynamics of the business systems. Many successful businesses intensively use simulation as an instrument for operational and strategic planning. The modeling methodology and simulation models of the business systems, as well as its validation, are described in (Kljajić et al., 2000). In comparison to the other methods, a dynamic analysis of the considered system behavior is the main advantage of testing the strategy with the aid of simulation scenarios (Larsen et al., 1997).

2. The warehouse model

2.1. Problem formulation

In this case we were dealing with a typical warehouse for storing products for further build in. The consumption of products depends on a production plan, which can be predicted with a certainty for six weeks. Lead time, for every product, is not variable. The problem occurs at defining the ordering quantity, because we have to consider the past orders and the variable consumption of a specific product. Long lead times also represent a problem, because they are usually much longer than the time period in which the production plan can be predicted with a certainty.
Four cases were analyzed with the methodology described below: case 1 (AX), case 2 (BX), case 3 (AY), case 4 (AY).

ABC classification divides products into three categories according to their value (importance), where products in category A have the highest value and products in category C have the lowest value. A different distribution of products offers XYZ classification, which divides products into three categories according to the dynamics of their consumption. The consumption for products in category X is stable and can be well predicted, while the consumption of products belonging to category Z is not stable and can not be predicted with any certainty.

So far, the company has used the ordering strategy (review period and order quantity) based on experiences of planners and their subjective assessments, without the use of optimization techniques.

The main goal of optimization was to rationalize warehouse ordering process, this means determining the interval between orders and the quantity to be ordered, so that the warehouse will operate with minimal common costs. Cost function includes:
- fixed ordering costs,
- transportation costs,
- costs of taking over the products,
- costs of physical storage,
- cost of capital.

The following limitations have to be taken into consideration:
- maximal warehouse capacity for a specific product must not be exceeded,
- no stockouts may occur.

From control point of view, our problem can be described with the advanced delayed difference equation:

\[ x(k + 1) = x(k) + d(k - \varphi(\tau_d)) - p(k), k = 0,1,2... \]  \hspace{1cm} (1)

where \( x(k) \), represents stock variable, \( d(k) \) material delivery and \( p(k) \) production process.

The delivery function \( d(k) \) is delayed for an average time \( \tau_d \) of an order \( o(k) \). Time delays are stochastic.

\[ d(k) = o(k - \varphi(\tau_d)) \]  \hspace{1cm} (2)

where \( \varphi(\tau_d) \) represents discrete uniform probability density function (pdf).

In order to compensate the stochastic delivery delay, the order policy \( o(k) \) has to be defined as:

\[ o(k) = f(x(k), d(k - \varphi(\tau_d)), p(k + \tau_p)) \]  \hspace{1cm} (3)
where $\phi(\tau_d)$ represents the stochastic time delay and $\tau_p$ the production plan horizon. It is necessary to find such $o(k)$ to minimize the following cost function:

$$J(o(k)) = \sum_{k=0}^{n} q(cx(k), ho(k)), k = 0,1,2..n$$

(4)

for $x_{\text{min}} \leq x \leq x_{\text{max}}$. In equation (4) $c$ and $h$ represents the cost of units of material on stock and its transportation.

Such stock control problems are very difficult to solve analytically. In order to improve the stock control problem, a simulation approach and the SD methodology have been chosen.

2.2. CLD of the warehouse model

Figure 1 represents the CLD from which the influences of the warehouse model elements can be observed. The arrow represents the direction of the influence and the + or – sign its polarity.

**Figure 1: Causal loop diagram of the warehouse model**

Delivery impacts Stock and Transportation Costs. If the amount of Delivery increases above what it would have been, the Stock and Transportation Costs are increased above the initial value. The increased value of Stock, increases Cost of physical storage and Cost of capital, but it decreases Ordering quantity. If the quantity of Production plan, which represents the reference value, is increased, Consumption and Ordering quantity are both increased. The increased value of Consumption decreases Stock. If the Ordering quantity is increased, the Delivery and Fix ordering cost are both increased. The increased values of Cost of physical storage and Cost of
capital increase the value of Holding cost, which increases the value of Total cost together with Fix ordering cost and Transportation cost.

There is one feedback loop in the causal loop diagram – negative feedback loop, which interconnects Stock, Ordering and Delivery and it represents the fact that we order less, if the stock level is high.

2.3. SD model of the warehouse

Figure 2 shows the warehouse simulation model built with Matlab (submodels are excluded). Matlab was chosen because it supports simulation with Simulink and offers a powerful computational engine, which provides a quick execution of the simulation runs.

Figure 2: The warehouse model built with Matlab

Two models were applied in order to find an ordering strategy which would produce less common costs: a model with fixed review period and a model with variable review period.

2.4. Models with different ordering strategies

According to the equations from (1) to (4), several strategies of stock control are considered, depending on the production planning period and its pdf, the delivery time delay and stock-on-hand. Stock level is changing according to the equation (1). The time point when an order is placed and the ordered quantity are based on equation (3), while considering the delivery time delay in equation (2) and the lowest value of cost function (4).

The first model (fixed review period) is based on making a sum of consumption for a specific product over a specific period (fixed) of time. The quantity of this sum is used in ordering quantity calculation together with the past orders and stock-on-hand. The specific period is fixed,
e.g. 7 weeks and it does not vary. This means that we place an order every 7 weeks. Figure 3 shows the principles of how this model works.

![Figure 3: Timeline of the model with a fixed review period](image1)

A production plan can be predicted with a certainty e.g. for 6 weeks. After this period, a production plan uncertainty factor (e.g. 3%) must be considered every 2 weeks. Therefore, a safety factor, which increases the ordering quantity, must be considered when placing an order (e.g. 10%).

If we assume that the lead time is 4 weeks, a review period is 7 weeks and a production plan can be predicted with a certainty for 6 weeks then we have a reliable prediction period of 2 weeks for the quantity to be ordered (6 – 4 = 2). For the rest of the review period (5 weeks; 7 – 2 = 5) a production plan is unreliable and a production plan uncertainty factor must be considered. This model is appropriate for products with great warehouse capacity.

The second model (variable review period) is also similar to the model with fixed review period. The difference is that the consumption for a specific product is not summed for a fixed review period – the consumption is summed until we reach the maximum warehouse capacity for a specific product. This model is presented in Figure 4 and is appropriate for products with very limited warehouse capacity.

![Figure 4: Timeline of the model with a variable review period](image2)
3. Results

The experiment was performed with the actual data provided by the observed company. To validate the simulation models, we used delivery and consumption data for every product for the period of three years. The company has confirmed the simulation inventory level dynamics based on the above mentioned data. They have also confirmed the validity of the costs the simulation model has calculated.

The model was changed in the "ordering" module to try out new ordering strategies. Monte Carlo simulation was used for variation of consumption unreliability. 50 simulation runs for every strategy on new simulation models were run for a year, using only consumption data. On the basis of these simulation runs, average costs and average stockouts were calculated. With several simulation runs and a calculation of average values, we have tried to minimize the influences of random generator, which represents the stochastic environment. Out of all simulation runs the maximum stock level was taken into consideration. The results of the simulation runs are presented in Figure 6, where the last strategy always represents the actual values provided by the observed company.

The stock level dynamics for new and old ordering strategies are shown in Figure 5. Although the new ordering strategy has reached much higher cumulative level than the old one, the new produced less common costs than the old one. This can be explained by the fact that the cost of physical storage and the capital costs represent only a small share regarding the common costs. As mentioned later, the transportation costs are the cost category which influences the common costs the most (Figure 7).

3.1. Case 1

Case 1 belongs to class A-X. Lead time for this product is 6 weeks, warehouse capacity is 9000 products. Warehouse capacity presents a serious limitation with this product. Simulation runs produced no stockouts. The warehouse capacity was exceeded at 3\textsuperscript{rd} and 4\textsuperscript{th} ordering strategies. The best strategy is the 5\textsuperscript{th} (variable review period), where common costs were reduced by 51\%. Transportation costs, which represent a great share of common costs, were
drastically reduced. Capital cost and ordering costs were also reduced, while take-over costs and physical storage costs increased.

3.2. Case 2

Case 2 belongs to class B-X. Lead time for this product is 4 weeks, warehouse capacity is 1500 products. A stockout occurred only at 2\(^{nd}\) ordering strategy, the warehouse capacity was not exceeded. The best strategy was 6\(^{th}\) (fix review period of 7 weeks), where common costs were reduced by 45%. All cost categories were reduced with the exception for physical storage costs.

3.3. Case 3

Case 3 belongs to class A-Y. Lead time for this product is 4 weeks, warehouse capacity is 120000 products. Stockouts occurred at 2\(^{nd}\), 3\(^{rd}\) and 4\(^{th}\) ordering strategy. The warehouse capacity was not exceeded. The best strategy is 7\(^{th}\) (fix review period of 8 weeks), where common costs were reduced by 35%. All cost categories were reduced with this strategy. Transportation costs and cost of capital were reduced the most.

Figure 6: Comparison of different simulation strategies for every product
3.4. Case 4

Case 4 belongs to class A-Y. Lead time for this product is 6 weeks, warehouse capacity is 10000 products.
Stockouts occurred at 1st, 5th, 6th, 7th ordering strategy. Capacity of the warehouse was not exceeded. The best strategy is 4th (fix review period of 5 weeks), where common costs were reduced by 18%. All cost categories were reduced with this strategy, except for the costs of physical storage. Transportation costs were reduced the most.

Figure 7 represents the average cost reducings achieved with the best ordering strategies for all four observed cases. Fixed ordering costs (reduced by 51%) and transportation costs (reduced by 40%) were drastically reduced because of less frequent ordering. Less frequent ordering produces less transportation and less ordering activities. The cost of capital (reduced by 45%) was another cost category drastically reduced. Such reduction is a result of an ordering strategy more adapted to the product consumption. The average cost of taking over the products decreased by 9%, while the cost of physical storage unexpectedly increased by 28%. Occurrence of this event will be researched in the future.

Figure 7: Average cost categories reducings and their shares regarding the common costs (1 – fixed ordering costs, 2 – costs of taking over the products, 3 – costs of physical storage, 4 – transportation costs, 5 – cost of capital)

Cost categories shares regarding common costs are also presented in Figure 7. The biggest share (74%) represent transportation costs, while fix ordering costs represent only a small portion of common costs (1%). As transportation costs represent the biggest share among all cost categories and it is of great significance that the new ordering strategies have reduced this cost category in average by 40%. We can conclude that the common costs for all products were reduced mostly by the decreased transportation costs.

4. Conclusion and discussion

This paper researched the warehouse stock optimization using two optimization algorithms. The SD approach was used in modeling and validation of the warehouse model. The simulation model was built using Matlab – Simulink. The observed company's representative cases were
analyzed. The company is using heuristic approach for seeking the best ordering policy. This approach could be improved as our research has shown significant cost reductions, without violating the two limitations: the warehouse capacity was not exceeded and no stockouts occurred.

Significant common cost reductions were achieved with simulation models for observed cases.: case 1 – 51%, case 2 – 45%, case 3 – 35%, case 4 – 18%. These improvements are certainly high and are the subject of further validation in the real case to confirm such benefits. However, according to these rather high improvements shown by simulation results, significant reduction in warehousing costs should be anticipated.

Common costs were reduced the most with products of class X, where future consumption can be predicted with a high certainty. Such cost reduction can be explained by high consumption predictability that allows accurate adaptation of the ordering strategy to consumption. Class Y consumption cannot be predicted so accurately and that can be the reason for the lower cost reductions in this class. The ordering strategies for this class have to consider a security factor, which increases the order quantity and that leads to higher inventory levels. The consequence is also a higher common cost.

These models can be easily implemented in companies, because their foundations are not complex mathematical formulas. For the order planner it is sufficient to know: lead time, inventory level, production plan, past orders, warehouse capacity and costs (ordering, transportation, take-over, physical storage, capital).

Presented simulation models were used for a validation of the warehousing process and as a preliminary study of the ordering process. Further studies will be conducted using fuzzy sets in order to improve ordering process. This paper presented a study of optimization of single products, leaving us the opportunity to optimize the whole warehousing process, including all products the observed company is storing, in the future research.

5. References


