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Dynamics of a system as a process of realization of its "potential".

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I. Introduction.

When describing the systems, irrespective of specific peculiarities of the case being considered and the concrete situation, we imagine them as a multitude of interrelated constituents, aspects, parameters, processes, stages, developments, etc. affecting each other. Let us call these component parts system «constituents». The systemic approach means that the investigated «object» is considered as a single whole where every «constituent» interacts with the other ones affecting them and being liable to the reciprocal direct or indirect impact. This interaction may be in a form of instantaneous impact of the system «constituents» on each other, and then the relevant interdependence is described by algebraic or differential equations. However, if there is a time delay between the impact of one «constituent» on the other and the feedback to the impact, then the interdependence is described by difference equations.

Thus the presence of feedbacks between the system «constituents» is described using a system of ordinary or (and) difference algebraic or (and) differential equations. We can assert the opposite as well:

If the system dynamics equations contain the parameters relating to different «constituents» systems, then the availability of such mathematical dependencies implies the existence of feedback between these «constituents».

The way of description of systems offered below may be referred to the class of «system dynamics» models, as the principal parameters are interrelated by the system of differential equations. However, our approach has one distinction from the usually considered models of this category. The distinction is as follows: the introduced parameters characterizing the system development are not the values to be directly observed. They may be regarded as indicators generally describing the key line and the tendencies of development of the system on the whole. Our approach may be called an abstract approach, for it may be applied for the most systems arising spontaneously in nature and in the society, irrespective of any specific features of their structure. In terms of this approach the feedback between the «constituents» of a particular system is mathematically expressed as a congruence of definitional domains of abstract values and interrelation of these values by differential equations. These values, being abstract indicators of the state of the system being considered, were designated as the «potential» and the «conditions of realization».

Talking of the state of a system one often uses the terms: "system potential" and "conditions of realization (or release!) of this potential". Analyzing, for example, the specific features of dynamics of different systems, one often states that "the potential" of a particular system is greater than of some other one, or one compares the "conditions for release of the potential" in different systems. This way one assumes implicitly that: 1) "the potential" and "the conditions" may be regarded as some numerical values 2) these values characterize the state of system at the utmost abstract level 3) these values may be obtained by some procedure of system information processing.

A question arises: is it possible to build a model of system development on the basis of exclusively of these two abstract values? This is what we have tried to do in this paper. We are not aware if such attempts have been made before. Our object was to reveal such peculiarities of system dynamics that can be found at this very abstract level of description.

It is known that the terms "potential" and "conditions of realization" are the categories of dialectical logic and are connected with each other by a complicated system of interrelations [2], [4]. We have tried to describe this logical interrelation using the mathematical methods. Here we have postulated a number of assumptions that express the sense of the offered terms.

Statement 1. Let F be "potential", U - "conditions for realization", $\{q_1, q_2, ..., q_N\}$- a set of registered system parameters that affect the abstract values F and U. Proceeding from the sense of the terms introduced, any change in the "conditions of realization" entails the change of the system "potential", that is, the values F and U are interrelated. There cannot be a situation when one of the values changes, while the other stays as it was. This interrelation of the values F and U means that no parameters exist that would affect only one value (F or U). These logical assumptions can be formalized in the form of the following statement:

The abstract values F and U are the functions of parameters $\{q_1, q_2, ..., q_N\}$:

$$U = U(q_1, q_2, ..., q_N); \quad F = F(q_1, q_2, ..., q_N). \qquad \textbf{(A)}$$

The congruence of the definitional domain of these functions is the mathematical expression of interdependence of the respective terms.

Statement 2. The registered parameters are the system data we have. For example, considering "a company" as a certain system we single out such parameters of this system as "the number of workers", "the company's assets", "the volume of operations", "the dept to other companies", etc. Each of these parameters somehow influences the values of "the conditions" and "the potential". This influence may be positive if the parameter growth results in the growth of F and (or) U, however it may be negative if the parameter growth is accompanied by the decrease of F and (or) U. In the above example the company's "potential" grows with the growth of the "company's assets", but the increase in the "amount of dept" lowers this "potential". Let us pose a question: does the growth of "assets" always mean the increase in the "potential", and the growth of "dept" – its reduction? We are apt to answer this question positively: yes, it is always so, with the exclusion of several non-standard situations. The same is true fore the other parameters – each of them affects the values F and U in quite certain way, expands or reduces them.

The mathematical expression of this fact is the monotony of functions $U = U(q_1, q_2, ..., q_N)$ and $F = F(q_1, q_2, ..., q_N)$ for each argument. **(B)**

Statement 3. The function of "potential" may be performed by any function of registered parameters that shows correctly the character of effect of these parameters on the "potential" value. Let us consider the space of these registered parameters $\{q_1, q_2, ..., q_N\}$. The surfaces of the fixed "potential" ("equipotential surfaces") are set by equations $F(q_1, q_2, ..., q_N) = Const$ in it. These equations express mathematically the way of affecting the "potential" value by particular parameters. They describe the mechanism, which makes the effect of one parameter be compensated by the effect of another one, having the potential value constant. That is why the equation $F(q_1, q_2, ..., q_N) = Const$ demonstrates the manner of connection of abstract value F with the registered system parameters. However, the numerical value of F as such may be different. It is just important for it to be the same for all the points of surface $F(q_1, q_2, ..., q_N) = Const$. Let $F(q_1, q_2, ..., q_N)$ be a function showing correctly the impact of the registrated parameters on the "potential" value. Then every non-decreasing function $\Psi(F)$ will have the same property. For that reason the choice of function, which is mathematical expression of «potential», is not ambiguous.

It means that the shift from the registered system parameters to the abstract value F is specified only to within the functional transformation $F \Rightarrow \Psi(F)$, where $\Psi(F)$ is an arbitrary non-decreasing function of F. **(C)**

This ambiguity lets us choose the function $F(q_1, q_2, ..., q_N)$ so as to simplify the type of functional dependence to the maximum extent. Let us consider, for example, a system described by parameters q_1 and q_2. Let the system "potential" be a strictly increasing function of these parameters. It is supposed that the impact (on the "potential" value) caused by decrease of one parameter twofold may be compensated by the increase of the other one threefold. It is evident that any non-decreasing function of argument $Q \equiv q_1 \cdot q_2^{log_3 2}$ as a "potential" may be taken. It might be, as an example, the function Q^n $(n > 0)$ or a^Q $(a > 0)$. However, it is more convenient to use the simplest version of functional dependence, being function $F(Q) = Q$ in this example. The choice is arbitrary and is accounted for by the considerations of simplication of calculations.

After we have identified the type of function $F(q_1, q_2, ..., q_N)$, we just have to determine what kind of «potential" we consider to be singular. The linear transformation does not change the type of functional dependence. Let us choose the linear transformation $\Psi(F) = \mu \cdot F$ so as $q_1 = q_2 = ... = q_N = 1$ "potential" were equal to a unity: $\tilde{F}(q_1 = 1; q_2 = 1; ...; q_N = 1) \equiv \Psi(F) \equiv \mu \cdot F(q_1 = 1; q_2 = 1; ...; q_N = 1) = 1$. This way we shall identify the unit of measurement for the "potential".

The "conditions" and the "potential" are interrelated and, as will be shown below, this ratio is mathematically described by the system of differential equations. Thus, by defining the form of functional dependence and "potential's" unit of measurement, we determine the form of functional dependence and unit of measurement for the "conditions". This means that the condition $F(q_1 = 1, q_2 = 1, ..., q_N = 1) = 1$, by which we remove the ambiguity of "potential's" function choice, is also a condition for the unequivocal choice of "conditions" function.

Statement 4. Let us introduce a new important term. Let us consider that depending on the number of "conditions" existing at that moment the system "potential" is realized either in full or in part. The realizable in a unit of time portion of the "potential" will be denoted as F_R.

The following ratio will be called **"the realization ratio":**

$$k = \frac{F_R}{F}. \quad (1)$$

We should make some comments to this definition. We shall designate the system "potential" release process as the "activity". The "activity is the **way of manifestation and the form of existence** of the realizable "potential". The realizable "potential" shows itself through the "activity" and exists only in the process of "activity". We judge of the value of realizable "potential" by the extent of its manifestation through the "activity". For example, the "enterprise activity" is a process of useful functioning of all of its component sections and is formed of the workers' "labor activity", the efficient operation of production facilities, the "managerial activity" on the part of administration, etc. The "activity" may be more or less intensive, depending on the "conditions" of its efficient realization. This enables us to specify the sense of the definition (1). The "realization ratio" shows which portion of the potential activity is represented by its realized part. If we define the "activity realized during the time Δt" as $A_R(\Delta t)$, and the maximum possible "activity" that could be realized during this time as $A(\Delta t)$, then the "realization ratio" is as follows:

$$k = \lim_{\Delta t \to 0} \frac{A_R(\Delta t)}{A(\Delta t)}. \quad (1^*)$$

The formulas (1) and (1*) are congruent in case of $F_R = \lim_{\Delta t \to 0} \frac{A_R(\Delta t)}{\Delta t}$ and $F = \lim_{\Delta t \to 0} \frac{A(\Delta t)}{\Delta t}$. For that reason the "potential" may be defined as the intensity of the "activity" or, which is the same, as the amount of «activity" that is realized (F_R) or may be realized (F) in a unit of time.

Proceeding from the sense of the terms introduced we shall postulate that "the realization ratio" is a function of "potential" and "conditions".

That is, the dependence of "realization ratio" on the parameters $\{q_1, q_2, ..., q_N\}$ *is expressed in the form:*

$$k(q_1, q_2, ..., q_N) = k(U(q_1, q_2, ..., q_N), F(q_1, q_2, ..., q_N)). \quad \textbf{(D)}$$

We shall postulate also the following properties of this function that reflect the logical interrelation of the terms earlier introduced:

1. There is an "optimal" number of "conditions" for any value of "potential", where the full realization of the system "potential" is achieved. That is,

$$\forall F, \exists U_0 : k(F, U_0) = 1. \quad (1.1)$$

2. The greater the "potential" the more "conditions" are needed for its full realization, that is, $U_0(F)$ - is a strictly increasing function. (1.2)

3. Having $U \neq U_0$; $0 < k(U, F) < 1$. (1.3)

4. Having $U \to 0$; $k(U, F) \to 0$, as nonzero "conditions" are necessary for the realization of system "potential". (1.4)

5. If the number of "conditions" is over the "optimal" U_0, then the system "potential" is realized only in part. Having $U \to \infty$; $k(U, F) \to m$; where m - then the least limit value that takes up the "realization ratio" in case of excess of available conditions. (1.5)

Statement 5. The "potential" dynamics is governed by the following peculiarities:

The process of realization of "potential" results in the growth of those potentialities of the system that are being realized in this process and in decrease of all the other non-realized potentialities. **(E)**

This law governs, for example, the abilities of living creatures: the abilities that are used grow with time, while; the abilities that are not exploited get atrophied. Another example: a company that functions grows and develops, while the company that has been stopped gets ruined. This principle reflects the very essence of life: only this thing grows that was able (took efforts) to grow.

Let us introduce the follow denotations:

$$\dot{U}(t) \equiv \frac{dU}{dt} = \lim_{\Delta t \to 0} \frac{\Delta U}{\Delta t}; \ \dot{F}(t) \equiv \frac{dF}{dt} = \lim_{\Delta t \to 0} \frac{\Delta F}{\Delta t}.$$

Let ΔF be the change of value of "potential" during Δt. This value consists of increment of realizable potencies ΔF_+ and diminution of unrealizable potencies ΔF_-. Let us denote the momentary increment (in a unit of time) of realizable potencies of a system $\dot{F}_+ \equiv \lim_{\Delta t \to 0} \frac{\Delta F_+}{\Delta t}$, and the momentary diminution (in a unit of time) of unrealizable potencies as $\dot{F}_- \equiv \lim_{\Delta t \to 0} \frac{\Delta F_-}{\Delta t}$ (both values are positive!). Then the complete change of value of "potential" in a unit of time is equal:

$$\dot{F} = \dot{F}_+ - \dot{F}_- \qquad (2)$$

It is evident that the more potencies we can realize the greater "potential" increment will be, that is the value $\dot{F}_+$ is a strictly increasing function of the realizable "potential" value. And on the contrary, the greater is the unrealized portion of the "potential" the greater is the diminution of unrealized "potencies" $\dot{F}_-$. Let us limit ourselves to the consideration of a simplest case of linear strictly increasing functions: $\dot{F}_+(F_R)$ and $\dot{F}_-(F - F_R)$.

$$\dot{F} = g_R \cdot F_R - g_D \cdot (F - F_R); \qquad (3)$$

$$g_R > 0; \ g_D > 0. \qquad (4)$$

Let $a \equiv g_D$; $b \equiv 1 + \frac{g_R}{g_D}$. (5)

Placing (1) in (3) we shall obtain the key **equation of "potential" dynamics as a linear approximation:**

$$\boxed{\dot{F} = a \cdot (b \cdot k(U, F) - 1) \cdot F}. \qquad (6)$$

Statement 6. The number of "conditions" changes due to two reasons:

1) The "potential" realization process is a process of consumption of available "conditions" and creation of new ones.

2) If the "potential" of a system is not realized the number of "conditions" available in the system decreases. **(F)**

The latter assumption is connected with the **universal physical principle of growth of entropy of systems** in which only natural mechanisms are in force. The growth of entropy, as is well known, illustrates the process of digression of the level of organization and orderliness of the system left on its own. This way, as an example, the abandoned houses decay and collapse, the fields not cultivated get overgrown with weeds, etc.

Let ΔU be the change of value of "conditions" during Δt. This value consists of two parts: the change due to the process of realization ("creation" minus "consumption": $\Delta U_+ - \Delta U_-$) and the change due to the growth of the system entropy.

Let us consider the first part. The more is the "potential" system realizable in a unit of time the obviously greater is the momentary increment of "conditions" in a unit of time ($\dot{U}_+ \equiv \lim\limits_{\Delta t \to 0} \frac{\Delta U_+}{\Delta t}$) and their expenditure ($\dot{U}_- \equiv \lim\limits_{\Delta t \to 0} \frac{\Delta U_-}{\Delta t}$). Assuming the linearity of interrelations again, we shall derive the first part of the "condition equation":

$$\dot{U}_{\mathrm{I}} = \dot{U}_+ - \dot{U}_- = x \cdot F_R - y \cdot F_R = n \cdot k(U,F) \cdot F \; ; \tag{7}$$

$$n = x - y \; ; \tag{8}$$

$$x > 0; \; y > 0 . \tag{8.1}$$

The second part of the "conditions equation" describes the decrease in the number of "conditions" due to natural processes of the system disorganization. The more is the available number of "conditions" the more is their loss in a unit of time. Being restricted again by simplest case of linear dependence, we get the following:

$$\dot{U}_{\mathrm{II}} = -l \cdot U \; ; \tag{9}$$

$$l > 0 . \tag{9.1}$$

Adding (7) and (9) we get to **the "conditions" dynamics equation as a linear approximation:**

$$\boxed{\dot{U} = n \cdot k(U,F) \cdot F - l \cdot U} \tag{10}$$

Equations (6) and (10) describe the evolution of the system as a process of changing its "potential" and "conditions of realization". Let us call them the **"evolution equations".** These represent non-linear first-order differential equations, which include the unknown subsidiary function – "realization ratio" that has some known functional properties (1.1)-(1.5). This function shows the specific reaction of the system to the lack or abundance of "conditions" in it. Due to the presence of the unknown function in the "evolution equations" it is impossible to get a strict analytical solution of this system of equations. However, knowing the functional properties of this function, it is possible to analyze the characteristic features of the solutions qualitatively. The analysis results in a number of interesting conclusions:

1. It follows from the equations (6) and (10) that the "realization ratio" does not depend on the choice of measurement units for abstract values F and U. Let us consider the "scaling" transformation of functions F and U ($F \Rightarrow \mu \cdot F$; $U \Rightarrow \mu \cdot U$), describing the change of unit of measurement for the "potential" and the "conditions". Here the left portions of the equations (6) and (10) are transformed in the following obvious way: $\dot{F} \Rightarrow \mu \cdot \dot{F}$ è $\dot{U} \Rightarrow \mu \cdot \dot{U}$, which would only be possible if the "realization ratio" does not depend on μ.

This means that *the "realization ratio" is a function of "conditions to potential ratio":* $k(U,F) \equiv k\left(\frac{U}{F}\right) \equiv k(Y)$. **(G)**

2. We may build a classification of possible "options of development" of system. The classification shows the character of change of (growth or diminution) of the three basic factors of the system: "potential", "conditions" and "conditions to potential ratio". 1) The increase or decrease of the "potential" evidences the **"progress" or "regress"** in the system development. 2) The increase or decrease of the "conditions" characterizes the **"effectivity" of development** (the development is "efficient" if the "potential system" realization does not result in decrease of "conditions" in it). 3) Finally, the "conditions to potential ratio" characterizes the extent of available of "conditions" for the "potential" (quantity "conditions" per the unit "potential"). If this value does not diminish we talk about the **"intensive development"**. Thus, for example, the development with the growing "potential" and "conditions", though with decrescent ratio of the second aspect to the first one, is classified as an "effective extensive progress". In this classification 6 options of system dynamics are possible. The "effective extensive regress" as well as "ineffective intensive progress" is not possible due to mathematical conjunction of indices underlying the classification. The classification of "development options" is given in **Table 1**.

3. Equation (6) and (10) include numerical parameters $a;b;n;l$ characterizing abstract properties of the system. The analysis of the equations shows that, having the same values of these parameters, different versions of development of the system are possible, as a rule. In other words, setting the parameters of a system is not equal to identifying the peculiarities of its dynamics. The particular "option of development" depends on the "conditions to potential ratio" at the start time. One and the same system may develop according to the "effective progress" option or "ineffective regress" option depending on the extent of availability of proper "conditions" for the "potential" at the start time.

4. Depending on the observance of a number of correlations between the parameters $a;b;n;l$ and m (1.5.) and the choice of function $k(U,F)$, all the systems may be divided into groups (or types) with identical factors (properties) of development. The classification of systems built on the basis of such division is discussed in **Cl.IV**. **Figures 8.1-8.9** illustrate some interesting peculiarities of development of the different types of system.

5. We may introduce the notion of **"area of development"** as an area of admissible values of "conditions to potential ratio". Thus we can see that normally several "areas of development" exist for every type of system and the system will develop within the "area" where it was at the start time. This means that availability of proper "conditions" for the "potential" will be confined by the limit values for this "area of development".

6. Let us consider the follow simple example. Let q_1 and q_2 are parameters of "constituents" of particular system. Let us consider the case of interaction of "constituents" in a form of instantaneous impact of this «constituents» on each other. The existence of such **feedback** between these «constituents» means that:

1) relations $q_1 = Q_1(q_2;\vec{p})$ and $q_2 = Q_2(q_1;\vec{p})$ exist ($\vec{p}$ is the vector of the other parameters of system);

2) functions $q_1 = Q_1(Q_2(q_1;\vec{p});\vec{p}) = \tilde{Q}_1(q_1)$ and $q_2 = Q_2(Q_1(q_2;\vec{p});\vec{p}) = \tilde{Q}_2(q_2)$ are not identities.

Let interrelation $U = \Phi(F)$ is deduced on the basis of equations (6) and (10). Let us relation $q_1 = Q_1(q_2;U)$ is deduced on the basis of dependence $U = U(q_1, q_2)$ and relation $q_2 = Q_2(q_1;F)$ is deduced on the basis of dependence $F = F(q_1, q_2)$. If the functions $U(q_1;q_2)$ and $\tilde{U}(q_1;q_2) \equiv \Phi(F(q_1;q_2))$ are different functions then formulas $q_1 = Q_1(Q_2(q_1;F);U)$ and $q_2 = Q_2(Q_1(q_2;U);F)$ are not identities and consequently feedback between the «constituents» of system exist.

In generally, in terms of our approach the feedback between the «constituents» q_i and q_k of a particular system is mathematically expressed by means of the follow two statements:

1) abstract values U and F are functions of parameters q_i and q_k;

2) determinant of Yakobi' matrix: $\begin{Bmatrix} \dfrac{\partial U}{\partial q_i} & \dfrac{\partial U}{\partial q_k} \\ \dfrac{\partial F}{\partial q_i} & \dfrac{\partial F}{\partial q_k} \end{Bmatrix}$ does not equal to zero.

These, as well as many other interesting peculiarities of system dynamics may be conveniently illustrated using the charts depicting the possible "options of system development" in the form of directed curves within the parameter plane (U, F) ("potential" (abscissa axis) – "conditions" (ordinate axis)). The development of the system is shown as a curve $(U(t), F(t))$ ("evolution curves") with each point corresponding to the state of the system at a particular moment of time (for example **Fig. 8.1-8.9)**.

II. The analysis of solutions of "evolution equations".

Let us introduce:

$$y \equiv \frac{U}{F}. \tag{11}$$

Using statement (**F**) and "evolution equations" (6) and (10), we get the following equation relative to variable y:

$$\frac{\dot{y}}{y} \equiv \frac{\dot{U}}{U} - \frac{\dot{F}}{F} = k(y) \cdot \left(\frac{\nu}{y} - a \cdot b \right) + (a - \lambda). \tag{12}$$

Solution of this equation is following:

$$\int \frac{dy}{\left(\dfrac{\nu - aby}{a - \lambda} \right) \cdot k(y) + y} = Const_1 + (a - \lambda) \cdot t. \tag{13}$$

The equations for functions $U(y)$ è $F(y)$ may be integrated like this:

$$LnU(y) + Const_2 = \int \frac{\left[\nu \cdot \dfrac{k(y)}{y} - \lambda \right] \cdot dy}{(\nu - aby) \cdot k(y) + (a - \lambda) \cdot y}; \tag{14}$$

$$LnF(y)+Const_2 = \int \frac{[b \cdot k(y)-1] \cdot a \cdot dy}{(\nu - aby) \cdot k(y) + (a-\lambda) \cdot y}. \qquad (15)$$

The constant present in the formulas (14) and (15) is identified by the initial "state" of the system.

Let us introduce the following subsidiary functions:

$$P_Y(y) \equiv (\nu - aby) \cdot k(y) + (a-\lambda) \cdot y; \qquad (16)$$

$$P_F(y) \equiv b \cdot k(y) - 1; \qquad (17)$$

$$P_U(y) \equiv \nu \cdot \frac{k(y)}{y} - \lambda. \qquad (18)$$

Using these functions, we can rewrite "evolution equations" and (12) as:

$$\dot{F} = a \cdot P_F(y) \cdot F; \qquad (6.1)$$

$$\dot{U} = P_U(y) \cdot U; \qquad (10.1)$$

$$\dot{y} = P_Y(y). \qquad (12.1)$$

It is obviously that functions $\dot{F}(t)$ and $P_F(y)$, $\dot{U}(t)$ and $P_U(y)$, $\dot{y}(t)$ and $P_Y(y)$ have similar (positive or negative) sign.

It is not difficult to see that:

$$P_Y(y) \equiv y \cdot (P_U(y) - a \cdot P_F(y)). \qquad (19)$$

If $y \to 0$, then $P_U(y) - a \cdot P_F(y) \to a - \lambda + \nu \cdot k'(0)$.

Then solution (13) – (15) can be represented in the following form:

$$\int \frac{(a-\lambda) \cdot dy}{P_Y(y)} = Const_1 + (a-\lambda) \cdot t \qquad (20)$$

$$LnU(y) + Const_2 = \int \frac{\cdot P_U(y) \cdot dy}{P_Y(y)} \qquad (21)$$

$$LnF(y) + Const_2 = \int \frac{P_F(y) \cdot a \cdot dy}{P_Y(y)} \qquad (22)$$

As can be seen from equation (20), if $t \to \pm\infty$, then $y \to y_Y^{(i)}$, where $y_Y^{(i)}$ is the root of equation: $P_Y(y) \equiv y \cdot (P_U(y) - a \cdot P_F(y)) = 0$. (23)

If $k'(0) \equiv \lim_{y \to 0} k'(y) \neq \infty$, then $y = 0$ is the root of equation (23). The remaining roots of the equation (23) comply with the ratio:

$$k(y) = \frac{(a-l) \cdot y}{aby - n} \qquad (24)$$

Let $y_Y^{(0)} = 0 < y_Y^{(1)} < y_Y^{(2)} < .. < y_Y^{(M)}$ are roots of equation (23) and let $y(0)$ is a ratio of quantity of «conditions" to value of "potential" at a start moment of time. It follows from the equation (20) that the whole "history" of system development takes place inside the domain: $y_Y^{(i-1)} < y < y_Y^{(i)}$, where $y_Y^{(i-1)}$ and $y_Y^{(i)}$ are two roots of equation (23) being "the closest" to value $y(0)$: $y_Y^{(i-1)} < y(0) < y_Y^{(i)}$. We denoted the domains $y_Y^{(i-1)} < y < y_Y^{(i)}$; $i = 1,2,...,M$ as the **"areas of development"** (**Fig.5**). Each "area of development" has a corresponding portion of plane (U, F), restricted by straight lines $U_{i-1}(F) = y_Y^{(i-1)} \cdot F$ and $U_i(F) = y_Y^{(i)} \cdot F$. As far as the "areas of development" are domains of term-constancy of function $P_Y(y)$, the character of monotony of function $y(t)$ within the "area of development" cannot change (formula (12.1)): the function $y(t)$ either grows or diminishes. So the "evolution curves" $(U(t), F(t))$ of every "area of development" are directed from one right line to the other: either from $U_{i-1}(F) = y_Y^{(i-1)} \cdot F$ to $U_i(F) = y_Y^{(i)} \cdot F$ or vice versa. In the first case the ratio of the number of "conditions" to the "potential" value grows tending to approach the limit value $y_Y^{(i)}$, and in the second case it diminishes tending to approach the limit value $y_Y^{(i-1)}$ (**Fig.6-7**).

Each "evolution curve" is a solution of "evolution equations" (6), (10). The "area of development" of system coincides with the "area of development" in which the system was at the start time. As the solution of "evolution equations" (20) – (22) depends only on the variable y and the system parameters $a; b; n; l$, the solutions pertaining to one and the same "area of development" are different only by the some multiplier whose value depends on the difference of original values U and F and (or) the shift of the countdown startup. The characteristic peculiarities of solutions for one and the same "area of development" are identical.

According to (6.1), (10.1), (12.1) the following is congruent for all solutions within one "area of development": 1) zeros of functions $\dot{F}(t)$ and $\dot{U}(t)$ (or $P_F(Y)$ and $P_U(Y)$); 2) domains of term-constancy of these functions.

This enables us to introduce the notion of **"option of development"** *as a totality of functions $U(t)$, $F(t)$ and $y(t)$ monotony properties.*

Each "option of development" is characterized by signs of three values: $\dot{Y}(t)$, $\dot{U}(t)$ and $\dot{F}(t)$.

Let us denote the following designations. The "area of development" being considered will be denoted as Ω_i ($y_Y^{(i-1)} < y < y_Y^{(i)}$), the totality of "areas of development" will be denoted as $\Omega = \{\Omega_1; ...; \Omega_M\}$, the "option of development" with *growing* functions $y(t)$, $U(t)$ and $F(t)$ will be denoted as $\Xi_{Y\uparrow;U\uparrow;F\uparrow} \equiv \Xi_{\uparrow\uparrow\uparrow}$ (Y is the first, U is the second, and F is the third index in the all formulas of this paper). The history of system's evolution may have several stages, each having its own specific "option of development".

For detailed description of the system evolution it is necessary to specify the "area of development" and the aggregate of "options of development", corresponding to the stages of evolution.

Say, the evolution consisting of two stages where at the first stage $U(t)$, $F(t)$ and $y(t)$ increase, and at the second stage $U(t)$ and $y(t)$ increase, while $F(t)$ decreases, is described by a set consisting of two "options": $\Xi_{\uparrow\uparrow\uparrow}$ and $\Xi_{\uparrow\uparrow\downarrow}$.

*The "**type of evolution**" is prescribed by specifying the "area of development" and the ordered set of "options of development" for the given "domain".*

Let us consider in more detail the mathematical conditions that determine the "area of development" of a system. These are the conditions of function $P_Y(y)$ term-constancy.

If $P_Y(y) > 0$, then the evolution curves of respective "area of development" comprise "options of development" with increasing function $y(t)$: $\Xi_{\uparrow\uparrow\uparrow}$; $\Xi_{\uparrow\uparrow\downarrow}$ and $\Xi_{\uparrow\downarrow\downarrow}$. Evolution curves are directed "counter-clockwise" if we proceed from the plane coordinates $(U\ ,F)$ origin (**Fig.5**).

If $P_Y(y) < 0$, then function $y(t)$ decrease and the evolution curves are directed «clockwise" (**Fig.5**). In this case the development may take place as three "options": $\Xi_{\downarrow\downarrow\uparrow}$; $\Xi_{\downarrow\downarrow\downarrow}$; $\Xi_{\downarrow\uparrow\uparrow}$.

The "options of development" $\Xi_{\uparrow\downarrow\uparrow}$; $\Xi_{\downarrow\uparrow\downarrow}$ are impossible due to identity: $\frac{\dot{y}}{y} \equiv \frac{\dot{U}}{U} - \frac{\dot{F}}{F}$.

For that reason there are exactly 6 "options of development": $\Xi_{\uparrow\uparrow\uparrow}$; $\Xi_{\uparrow\uparrow\downarrow}$; $\Xi_{\uparrow\downarrow\downarrow}$; $\Xi_{\downarrow\downarrow\uparrow}$; $\Xi_{\downarrow\downarrow\downarrow}$; $\Xi_{\downarrow\uparrow\uparrow}$. Classification of these basic "options" is represented in **Table 1**.

The plane $(U\ ,F)$ is spit into several "areas of development", with each having a specific sign (positive or negative) of function $P_Y(y)$. The "areas" having one common border have different respective function $P_Y(y)$ signs. In the "areas" $P_Y(y) > 0$ the evolution curves are aimed at the top border of the "area", in the "areas" $P_Y(y) < 0$ - they are directed to the bottom border of the "area". For that reason the "area" borders are the lines to which the evolution curves tend to converge or from which they disperse. In the "areas" $P_Y(y) > 0$ the top border is the **"line of convergence"** of evolution curves, while the bottom border is the **"line of dispersion" (Fig.7).** With time all the evolution curves line up along the "convergence lines", that is, along certain directions of the plane $(U\ ,F)$. The "convergence lines" and "dispersion lines" alternate. Each line is prescribed a certain value y. The value $y_{+\infty} \equiv \lim_{t\to+\infty} y(t)$ - is the value y for the "convergence lines", while the value $y_{-\infty} \equiv \lim_{t\to-\infty} y(t)$ - is the value y for "dispersion lines" of evolution curves.

The set of systems having the different values y at start time transforms during the time into the set of systems having only some certain values y. Consequently the initial disordered distribution of systems on the plane $(U\ ,F)$ transforms into the ordered distribution of systems on several groups with certain values y. We can consider this property of evolution as one from many possible form of creation of order out chaos. The problem of self-organization of systems in evolution process is broadly discussed in modern literature (see for example, [3], [5]-[8], [11]). Our approach allows connecting self-organization with universal principles of evolution.

The values y possible for every "area" are in accord with one of the inequations: 1) $y_{-\infty} < y < y_{+\infty}$ for "areas" $P_Y(y) > 0$ and 2) $y_{+\infty} < y < y_{-\infty}$ for "areas"

$P_Y(y)<0$. The "areas" of the first type will be denoted as Ω^+, and the "areas" of the second time as Ω^-. The limit values $y_{-\infty}$ and $y_{+\infty}$ for these "areas" will be denoted as $y_{-\infty}(\Omega^+)$; $y_{-\infty}(\Omega^-)$; $y_{+\infty}(\Omega^+)$; $y_{+\infty}(\Omega^-)$. The "areas" having the common border are, probably, the "areas" of different types: Ω^+ and Ω^-. It is not hard to see that for the bordering "areas" one of the equations holds true: 1) either $y_{-\infty}(\Omega^+)\equiv y_{-\infty}(\Omega^-)$; 2) or $y_{+\infty}(\Omega^+)\equiv y_{+\infty}(\Omega^-)$. In the first case the border is a "dispersion line", in the second case it is the "convergence line" of evolution curves. We may formulate a simple rule setting the "status" of the border.

The top border of "areas" Ω^- is the "dispersion line", the bottom border is the "line of convergence". And contrary for the "areas" Ω^+.

Let $\Omega=\{\Omega_1;...;\Omega_M\}$ be a set of "areas of development" for a system. Two options of alternating of function $P_Y(y)$ relators (signs) are possible: 1) $\Omega^I=\{\Omega_1^-;...;\Omega_M^{(-1)^M}\}$ and 2) $\Omega^{II}=\{\Omega_1^+;...;\Omega_M^{(-1)^{M+1}}\}$. In the first case the inequation $a-\lambda+\nu\cdot k'(0)<0$ takes place. In the second case the term of inequality is reversed.

The borders of "areas of development" are roots of equation:

$$P_Y(y)\equiv y\cdot(P_U(y)-a\cdot P_F(y))\equiv(n-aby)\cdot k(y)+(a-l)\cdot y=0. \tag{23}$$

The number of positive roots of equation (24) depends on choice of function $k(y)$ and on values $\{a;b;n;l\}$.

Let us sum up the results we obtained.

Each value y has a corresponding right line in the plane (U,F) passing through the coordinate origin (the tangent of angle of inclination of this line is equal to y). The dependence $y(t)$ (formula (20)) describes the rotation of this straight line around the coordinate origin in a clockwise direction if $y(t)$ diminishes and counter-clockwise if $y(t)$ gains in value. The "evolution curve" is described uniquely by formulas (20)–(22), if the original values $U(t=0)$ $F(t=0)$ and parameters $\{a;b;n;l\}$ are assigned.

The factors accounting for the external look of the "evolution curve":

1) The zeros of function $P_Y(y)\equiv(n-aby)\cdot k(y)+(a-l)\cdot y$ determine the system's "area of development". The number of "conditions" fitting a single "potential", that is, the value $y=\dfrac{U}{F}$, will increase if we have $P_Y(y)>0$ and will decrease if we have $P_Y(y)<0$.
2) The function $P_F(y)\equiv b\cdot k(y)-1$ relator (sign of value of this function) prescribes if the system "potential" will grow ($P_F(y)>0$) or diminish ($P_F(y)<0$).
3) The function $P_U(y)\equiv n\cdot\dfrac{k(y)}{y}-l$ relator prescribes if the number of "conditions" will grow ($P_U(y)>0$) or diminish ($P_U(y)<0$).

All possible "options of development" of the system thus may be divided into 6 groups depending on functions $P_U(y)$; $P_F(y)$; $P_Y(y)$ relator. The classification of "options of development" designed on the basis of this division is contained in **Table. I**.

Let us introduce the following subsidiary functions (**Fig.1-3**):

1) $F_F(y)=\dfrac{1}{b}$; 2) $F_U(y)=\dfrac{\lambda}{\nu}\cdot y$; 3) $F_Y(y)=\dfrac{(a-\lambda)\cdot y}{aby-\nu}$.

1) Let us denote the positive roots of equation $F_F(y)=k(y)$ as $y_F^{(1)};y_F^{(2)};...$(there is always at least one root) (**Fig.1**). (25)

2) Let us denote the positive roots of equation $F_U(y)=k(y)$ as $y_U^{(1)};y_U^{(2)};...$ (if $n<0$, then the equation has no roots) (**Fig.2**). (26)

3) Let us denote the positive roots of equation $F_Y(y)=k(y)$ as $y_Y^{(1)};y_Y^{(2)};...$ (**Fig.3**). (27)

$y_F^{(1)};y_F^{(2)};...;$ $y_U^{(1)};y_U^{(2)};...;$ $y_Y^{(1)};y_Y^{(2)};...$ points split the multitude of permissible $0<y<\infty$ values into areas corresponding to one of the system's possible "options of development". The development of the system takes place within one of the "areas of development" $\left(y_Y^{(i)};y_Y^{(i+1)}\right)$. The roots $y_F^{(1)};y_F^{(2)};...$ and $y_U^{(1)};y_U^{(2)};...$ appurtenant to "areas" $\left(y_Y^{(i)};y_Y^{(i+1)}\right)$ divide in into sub-areas, where which sub-area is characterized by a certain function $P_F(y)$ and $P_U(y)$ relators, that is, conforming to a specific "option of development" of the system.

The **charts 4-6** illustrate the described regularities using a concrete example.

III. External impacts as the reason of change in the system's "area of development".

The development of the system described by evolution equations does not withdraw the system beyond the "area of development" in which the system was at the original moment of time. Let us consider the transition of the system from one "area of development" to the other. Such transition is possible if the value y changes in discrete steps either for account of increase in number of "conditions" or the abrupt decrease of the system "potential". In both cases the system is affected by the "external" impact resulting in the change of "area of development" of the system. The new "area" is compliant with the new "type of evolution" and the new limit value y. One may suppose that *the change of "area of development" is one of the methods of withdrawing the system from crisis*.

For example, the crisis in company activities is overcome by: 1) creation of additional "conditions" of development (loans, credit, deferral of payments, etc.), 2) reduction of system "potential" (firing the workers, cut in volume of production, sale of some property, etc.).

We may denote the shift of the system from "area" Ω_i to "area" Ω_k ($\Omega_i\to\Omega_k$) "antirecessionary" if the following conditions are met:

1) The final “option of development” in the new “area” Ω_k is characterized by the growth of values of “potential” and “conditions” ($\uparrow\uparrow\uparrow$ or $\downarrow\uparrow\uparrow$); (28)

2) $y_{+\infty}(\Omega_k) > y_{+\infty}(\Omega_i)$. (29)

Let us consider the issue of the extent of external impact on the system. Let ΔU and ΔF be absolute increments for the values U and F that are necessary for transition to the new “area of development”. Let U_0 and F_0 be the values of U and F prior to the transition, Ω_i being the original “area of development”, Ω_k being the “area of development” of the system after the shift. Let us consider the two ways of transition: 1) $\Delta U = 0$ è 2) $\Delta F = 0$.

Let us consider for example the case $\uparrow\uparrow\uparrow$ represented in **Fig.7**. Then $\Omega_k \equiv \Omega_k^+$.

1) In the first way of transition the increment ΔF necessary for the shift to the new “area” is set by the inequation: $\Delta F > F_0 - \dfrac{U_0}{y_{-\infty}(\Omega_k)}$. (30)

2) In the second case ($\Delta F = 0$) the follow inequation must hold true: $\Delta U > F_0 \cdot y_{-\infty}(\Omega_k) - U_0$. (31)

If the increment of the number of “conditions” is limited by some value $\delta U < F_0 \cdot y_{-\infty}(\Omega_k) - U_0$, then, to have the transition, it is necessary to decrease the “potential” by value $\Delta F > F_0 - \dfrac{U_0 + dU}{y_{-\infty}(\Omega_k)}$. (32)

IV. Classification of systems on the basis of its “evolution properties”.

Let us arrange the roots of equations (25)-(27) ($y_F^{(1)}; y_F^{(2)};...;$ $y_U^{(1)}; y_U^{(2)};...;$ $y_Y^{(1)}; y_Y^{(2)};...$) in ascending order and number them. Let us denote as $y_{A(i)}^{(k)}$, where $A(i) = \{Y; U; F\}$, $i = 1,2,3$, and the upper index is the new root number. Each system is characterized by certain division of set $y > 0$ into subsets $y_{A(i)}^{(k)} < y < y_{A(n)}^{(m)}$. Each such subset conforms to a certain “option of development”.

Then sequence of “options of development” characterizes the “evolution properties” of a system.

The systems with similar “evolution properties” will be called one-type systems. Each type of systems conforms to a certain method of decomposition of set $y > 0$ into subsets $y_{A(i)}^{(k)} < y < y_{A(n)}^{(m)}$.

To describe the method of decomposition it is sufficient to list the “options of development” of the system with ascending y.

For example, the case pictured in **Fig.7** is described by the following set of “options”: $\Xi_{\uparrow\downarrow\downarrow}; \Xi_{\uparrow\uparrow\downarrow}; \Xi_{\uparrow\uparrow\uparrow}; \Xi_{\downarrow\uparrow\uparrow}; \Xi_{\downarrow\downarrow\uparrow}; \Xi_{\downarrow\downarrow\downarrow}; \Xi_{\uparrow\downarrow\downarrow}$.

In general each system type has a corresponding ordered set of "options". As far as in the area $0 < y < y_{A(n)}^{(1)}$ the "potential" diminishes, only $\Xi_{\uparrow\uparrow\downarrow}$; $\Xi_{\uparrow\downarrow\downarrow}$; $\Xi_{\downarrow\downarrow\downarrow}$ "options" are possible in this "area of development".

Let us choose of all ordered sets composed of 6 possible "options" ($\Xi_{\uparrow\uparrow\uparrow}$; $\Xi_{\uparrow\uparrow\downarrow}$; $\Xi_{\uparrow\downarrow\downarrow}$; $\Xi_{\downarrow\downarrow\uparrow}$; $\Xi_{\downarrow\downarrow\downarrow}$; $\Xi_{\downarrow\uparrow\uparrow}$) those corresponding to the "evolution equations".

It is not difficulty to see, that all the possible 6 "options of development" can be deduced from 3 basic "options": $\Xi_{\uparrow\uparrow\uparrow}$, $\Xi_{\uparrow\uparrow\downarrow}$, $\Xi_{\uparrow\downarrow\downarrow}$ by time inversion operation. Let us introduce the following designations for the basic "options": $3 \equiv \Xi_{\uparrow\uparrow\uparrow}$, $2 \equiv \Xi_{\uparrow\uparrow\downarrow}$, $1 \equiv \Xi_{\uparrow\downarrow\downarrow}$. Let us designate the operation of time inversion by upper line $\overline{\Xi}$. The following relations take place: $\overline{3} \equiv \Xi_{\downarrow\downarrow\downarrow} = \overline{\Xi}_{\uparrow\uparrow\uparrow}$; $\overline{2} \equiv \Xi_{\downarrow\downarrow\uparrow} = \overline{\Xi}_{\uparrow\uparrow\downarrow}$; $\overline{1} \equiv \Xi_{\downarrow\uparrow\uparrow} = \overline{\Xi}_{\uparrow\downarrow\downarrow}$.

Let us consider the simplest case:

1) We shall consider that function $k(y)$ is convex in the area of increment (if $k'(y) > 0$, then $k''(y) < 0$) and function $k(y)$ has only one maximum point. (*)

2) Let all the roots of equations $F_F(y) = k(y)$, $F_U(y) = k(y)$ and $F_Y(y) = k(y)$ be different and the functions $F_Y(y)$, $F_U(y)$ have no points of contact with function $k(y)$ (only points of intersection are observed). (**)

Having these assumptions is not difficult to list all types of systems having one and two "areas of development". Let us consider the following alternatives:

1.1) Equation $F_F(y) = k(y)$ has two roots ($\frac{1}{b} > m$). There are three areas of term-constancy of function $\dot{F}(t)$ conforming to the "options of development": $\Xi_{**\downarrow}$; $\Xi_{**\uparrow}$; $\Xi_{**\downarrow}$, (**Fig.1**);

1.2) Equation $F_F(y) = k(y)$ has one root ($\frac{1}{b} < m$). There are two areas of term-constancy of function $\dot{F}(t)$ conforming to the "options of development": $\Xi_{**\downarrow}$; $\Xi_{**\uparrow}$ (**Fig.1**);

2.1) Equation $F_U(y) = k(y)$ has no roots. Function $U(t)$ diminishes over the whole area $y > 0$. Consequently, there is only one possibility - $\Xi_{*\downarrow*}$ (**Fig.2)**;

2.2) Equation $F_U(y) = k(y)$ has one root. The sequence of "options of development" is the following: $\Xi_{*\uparrow*}$, $\Xi_{*\downarrow*}$ (**Fig.2)**;

3.1) Equation $F_Y(y) = k(y)$ has no roots. Function $Y(t)$ diminishes over the whole area $y > 0$. There is only one "options of development" - $\Xi_{\downarrow**}$ (**Fig.3**);

3.2) Equation $F_Y(y) = k(y)$ has at least one root (**Fig.3**).

Let us consider the follow four possibilities: (1.1 and 2.1), (1.1 and 2.2), (1.2 and 2.1), (1.2 and 2.2):

(1.1 and 2.1) $\Xi_{*\downarrow\downarrow}$; $\Xi_{\downarrow\downarrow\uparrow}$; $\Xi_{*\downarrow\downarrow}$, (À11)

(1.1 and 2.2) Three options are possible:

à) $\Xi_{\uparrow\uparrow\downarrow}$; $\Xi_{*\downarrow\downarrow}$; $\Xi_{\downarrow\downarrow\uparrow}$; $\Xi_{*\downarrow\downarrow}$, (À12à)

b) $\Xi_{\uparrow\uparrow\downarrow}$; $\Xi_{*\uparrow\uparrow}$; $\Xi_{\downarrow\downarrow\uparrow}$; $\Xi_{*\downarrow\downarrow}$, (À12b)

ñ) $\Xi_{\uparrow\uparrow\downarrow}$; $\Xi_{*\uparrow\uparrow}$; $\Xi_{\uparrow\uparrow\downarrow}$; $\Xi_{*\downarrow\downarrow}$, (À12c)

(1.2 and 2.1) $\Xi_{*\downarrow\downarrow}$; $\Xi_{\downarrow\downarrow\uparrow}$, (À21)

(1.2 and 2.2) Two options are possible:

à) $\Xi_{\uparrow\uparrow\downarrow}$; $\Xi_{*\downarrow\downarrow}$; $\Xi_{\downarrow\downarrow\uparrow}$, (À22à)

b) $\Xi_{\uparrow\uparrow\downarrow}$; $\Xi_{*\uparrow\uparrow}$; $\Xi_{\downarrow\downarrow\uparrow}$, (À22b)

1. Let equation $F_Y(y) = k(y)$ has no roots. Then there is only one "area of development". Only cases (À11), (À21) are possible:

À110. $3;\ \overline{2};\ \overline{3}$ ($\Xi_{\downarrow\downarrow\downarrow}$; $\Xi_{\downarrow\downarrow\uparrow}$; $\Xi_{\downarrow\downarrow\downarrow}$);

À210. $3;\ \overline{2}$ ($\Xi_{\downarrow\downarrow\downarrow}$; $\Xi_{\downarrow\downarrow\uparrow}$);

2. Let equation $F_Y(y) = k(y)$ has one root. In this case there are two "areas of development". Function $y(t)$ increases into the first "area" but it decreases into the second "area". Only seven types of systems are possible:

A111. $1;\ \overline{3};\ \overline{2};\ \overline{3}$ ($\Xi_{\uparrow\downarrow\downarrow}$; $\Xi_{\downarrow\downarrow\downarrow}$; $\Xi_{\downarrow\downarrow\uparrow}$; $\Xi_{\downarrow\downarrow\downarrow}$);

A12a. $\overline{2};\ 1;\ \overline{3};\ \overline{2};\ \overline{3};$

A12b. $2;\ 3;\ \overline{1};\ \overline{2};\ \overline{3};$

A12c. $2;\ 3;\ 2;\ 1;\ \overline{3};$

A21 $1;\ \overline{3};\ \overline{2};$

A22a. $2;\ 1;\ \overline{3};\ \overline{2};$

A22b. $2;\ 3;\ \overline{1};\ \overline{2};$

So all systems (with properties (*) and (**)) having one "area of development" are dvided into 2 types, while the systems with two "areas of development" are divided into 7 types (**Fig. 8.1 – 8.9**).

Classification of systems having other number of "areas of development" can be considered analogously.

V. The systems comprising several subsystems.

So far we have considered particular systems as one whole. But as a rule any system is an agglomeration of a number of interacting subsystems. The number of subsystems forming the system and the mode of their interaction condition the «structure» of the system.

Let us consider system comprising several subsystems. Let F_i and U_i, $i = 1,2,\ldots,N$ be the "potential" and "conditions" of subsystems, F and U be the "potential" and "conditions" of the composite system.

Let us call the systems to be **«identical»** if they meet the three following conditions:

1) The function $k(U,F)$ for all systems is the same.
2) The parameters a, b, ν, λ of the systems are identical.
3) The initial conditions for all systems are identical. ($F_1(0) = F_2(0) = \ldots = F_N(0)$, $U_1(0) = U_2(0) = \ldots = U_N(0)$).

The systems meeting conditions 1) and 2) only will be designated as **«similar»** systems.

To describe in mathematical terms the operation of integration of several «identical» systems in one, let us assume the following:

Statement 7. *The value of «potential» («conditions») in the system comprising several «identical» subsystems is equal to the sum of values of «potentials» («conditions») of these subsystems:* $F = \sum_{i=1}^{N} F_i$, $U = \sum_{i=1}^{N} U_i$. **(H)**

The assumption **(H)** is the simplest and most natural mathematical formula of the logical ratio «to consist of …». The assumption **(H)** is correct only for the «identical» systems and, as we can see further, it must be specified in case of «similar non-identical» systems.

The «evolution equations» express the universal peculiarities of systems development. They must be correct for any systems spontaneously arising in nature and in the society. Therefore the «evolution equations» must be executed both for the system being considered and its components-subsystems. However, *due to non-linearity of «evolution equations», the «potential» and «conditions» of the composite system can be not equal to the sum of respective values of the «subsystems».*

Let us consider the system comprising two “similar” subsystems. Let F_1, U_1 and F_2, U_2 are the values of “potential” and “conditions of realization” in subsystems and let F, U are the values of “potential” and “conditions of realization” in the composite system:

$$\dot{F}_1 = a \cdot (b \cdot k(U_1, F_1) - 1) \cdot F_1; \qquad \textbf{(I)}$$
$$\dot{U}_1 = \nu \cdot k(U_1, F_1) \cdot F_1 - \lambda \cdot U_1$$

$$\dot{F}_2 = a \cdot (b \cdot k(U_2, F_2) - 1) \cdot F_2; \qquad \textbf{(II)}$$
$$\dot{U}_2 = \nu \cdot k(U_2, F_2) \cdot F_2 - \lambda \cdot U_2;$$

$$\dot{F} = A \cdot (B \cdot K(U, F) - 1) \cdot F; \qquad \textbf{(III)}$$
$$\dot{U} = \Gamma \cdot K(U, F) \cdot F - \Lambda \cdot U$$

where A, B, Γ, Λ are parameters of composite system and $K(U,F)$ is “realization ratio” in this system.

We shall prove the following:

1°. Functions $K(U,F)$ and $k(U,F)$ are identical.

2°. The parameters A, B, Γ, Λ of the composite system are equal to the respective parameters of a, b, ν, λ of the subsystems.

The substantiation:

1°. Let the «potentials» and «conditions» of the subsystems are equal at the initial moment: $F_1(0) = F_2(0) = f(0)$ è $U_1(0) = U_2(0) = u(0)$. Then they will be equal with any other $t > 0$: $F_1(t) = F_2(t) \equiv f(t)$, $U_1(t) = U_2(t) \equiv u(t)$.

In this case the “similar” systems are «identical», and the assumption **(H)** holds true. In this case both sets of equations **(I)** and **(II)** are reduced to the following sets:

$$\dot{f} = a \cdot (b \cdot k(u, f) - 1) \cdot f \,, \tag{33}$$

$$\dot{u} = \nu \cdot k(u, f) \cdot f - \lambda \cdot u \,. \tag{34}$$

With regard for the property of homogeneity **(G)** of function $k(U,F)$ we may reproduce the following chain of equalities:

$$F = \frac{F^{(R)}}{K(U,F)} = \frac{F^{(R)}}{K(2u,2f)} = \frac{F^{(R)}}{K(u,f)} = F_1 + F_2 = \frac{F_1^{(R)} + F_2^{(R)}}{k(u,f)} = \frac{F^{(R)}}{k(u,f)} \,. \tag{35}$$

Statement 1° follows from (35).

$$K(u, f) = k(u, f) \,. \tag{36}$$

2°. Substituting $\dot{U} = \dot{U}_1 + \dot{U}_2 = 2 \cdot \dot{u}$ and $\dot{F} = \dot{F}_1 + \dot{F}_2 = 2 \cdot \dot{f}$ in **(III)** and keeping (36) in mind we derive:

$$\dot{f} = A \cdot (B \cdot K(2u, 2f) - 1) \cdot f = A \cdot (B \cdot k(u, f) - 1) \cdot f \,; \tag{37}$$

$$\dot{u} = \Gamma \cdot K(2u, 2f) \cdot f - \Lambda \cdot u = \Gamma \cdot k(u, f) \cdot f - \Lambda \cdot u \,. \tag{38}$$

Comparing (37)-(38) and (33)-(34) we derive the following:

$$A = a \,,\; B = b \,,\; \Gamma = \nu \,,\; \Lambda = \lambda \,. \tag{39}$$

As far as sets of equations **(I)–(III)** must hold true for any choice of initial conditions, the equalities (36) and (39) are correct as well for the case of «similar» «non-identical» systems. Using the induction, this argument may apply as well to the situation of arbitrary number of «similar» systems.

Finally we come to the following important results:

1) The parameters of the system comprising the arbitrary number of «similar» subsystems are equal to the corresponding parameters of the subsystems 2) The function of «realisation ratio» of such composite system is congruent with the function of «realisation ratio» of the subsystems.

Let us consider the case of two "similar non-identical" systems. The composite system is described by the follow set of equations:

$$\begin{aligned} \dot{F} &= a \cdot (b \cdot k(U, F) - 1) \cdot F \\ \dot{U} &= \nu \cdot k(U, F) \cdot F - \lambda \cdot U \end{aligned} \tag{40}$$

If $F = F_1 + F_2$ and $U = U_1 + U_2$, then the following correlation must hold true:

$$k(U, F) = \frac{F_1}{F_1 + F_2} \cdot k(U_1, F_1) + \frac{F_2}{F_1 + F_2} \cdot k(U_2, F_2) \,. \tag{41}$$

Let us introduce variables: $y \equiv \dfrac{U}{F} = \dfrac{U_1 + U_2}{F_1 + F_2}$, $y_1 \equiv \dfrac{U_1}{F_1}$, $y_2 \equiv \dfrac{U_2}{F_2}$. (42)

Let $y_2 > y_1$. Let us introduce the following denotation:

$$\frac{y_2 - y}{y_2 - y_1} = \frac{F_1}{F_1 + F_2} \equiv \alpha \,; \tag{43}$$

It is not difficulty to see that:

$$\frac{y-y_1}{y_2-y_1}=\frac{F_2}{F_1+F_2}=1-\alpha\,; \qquad (44)$$

$$y=\alpha\cdot y_1+(1-\alpha)\cdot y_2\,. \qquad (45)$$

In these designations the correlation (41) is congruent with the condition of function $k(U,F)$ linearity:

$$k(\alpha\cdot y_1+(1-\alpha)\cdot y_2)=\alpha\cdot k(y_1)+(1-\alpha)\cdot k(y_2)\,. \qquad (46)$$

However function $k(y)$ is not the linear function and therefore:

$F\neq F_1+F_2$ and $U\neq U_1+U_2$.

Let $F=F_1+F_2+F_{12}$ and $U=U_1+U_2+U_{12}$, (47)

where F_{12} and U_{12} are the additional items introduced with the purpose to ensure compatibility of the three sets of equations: **(I), (II)** and (40). They may be regarded as the terms describing the «interaction» of subsystems within the composite system. It is remarkable that *the composite system has some additional opportunities of development connected with the interaction of its subsystems*. The availability of these new possibilities of development is expressed mathematically by introduction of additional summands: F_{12} and U_{12}. The contribution of these summands in the "potential" ("conditions") value may be interpreted as the impact of the system's «structure» on its "potential" ("conditions"). So let us call them the "structural summands (or "structural terms)".

The "structural terms" comply with the following equations:

$$\dot{U}_{12}=\nu\cdot(F_1+F_2)\cdot S(y,y_1,y_2)+\nu\cdot k(y)\cdot F_{12}-\lambda\cdot U_{12}\,, \qquad (48)$$

$$\dot{F}_{12}=ab\cdot(F_1+F_2)\cdot S(y,y_1,y_2)+a\cdot(bk(y)-1)\cdot F_{12}\,, \qquad (49)$$

$$S(y,y_1,y_2)=k(y)-\alpha\cdot k(y_1)-(1-\alpha)\cdot k(y_2)\,. \qquad (50)$$

Let us call the expression (50) as S–factor (structural factor). If $|y_2-y_1|\to 0$, then inequation $S(y,y_1,y_2)>0$ is the condition of convexity of function $k(y)$ in point y: $k''(y)<0$. Let inequation $k''(y)\neq 0$ take place in the whole area $y_1\leq y\leq y_2$ (function $k(y)$ has no points of inflection). If $S(y,y_1,y_2)>0$ ($S(y,y_1,y_2)<0$), then function $k(y)$ is convex (concave) in this area.

Let at some moment of time t_0 a new composite system emerge as a result of integration of two "similar" systems. Prior to this moment the "interaction" between the systems was absent and therefore $F_{12}(t)=0$ and $U_{12}(t)=0$ during $t\leq t_0$. Due to continuity of functions $F_{12}(t)$ and $U_{12}(t)$ we may neglect the "structural terms" F_{12} and U_{12} in the right part of the formulas (48)-(49) when considering the initial stage of formation of the new system. For that reason the change of "structural terms" at the first stage of formation of composite systems is determined only by the S-factor. Therefore if function $k(y)$ is convex and $\nu>0$ then the result of the system integration will be represented by growth of "potential" and "conditions". As can be seem from **Fig.1**, the function $k(y)$ is convex only within the limited area of values. Hence an important conclusion:

The unification of several systems in one results in the growth of "potential" and "conditions" only when the number of "conditions" matching a single "potential" is not too small, however, not exceeding some certain value.

The growth of "potential" of the composite system means that value of "potential" depends on the structure of system. This growth is the result of application of universal evolution principles to description of evolution of the composite system. Existence of the structure means existence organization (order) in the composite system. *Just organization* of simple systems in one composite system transforms disordered set of systems into the *new system*. Consequently unification of several systems in one is not formal procedure. The universal principles of evolution can work if only the process of unification of some systems in one results in creation of some structure in the composite system. Consequently self-organization of the system can be understood as the result of existence of two facts: 1) universal principles of evolution, 2) unification of some systems in one. Extensive literature devoted to this item exists (see, for example survey in paper [1]).

If we consider particular persons as systems, then one of the principal indicators showing the number of "conditions" per one "potential" is the income per capita in a family. The field within which the unification of systems results in the growth of "potential" may be characterized, using this indicator, as "stably low and medium income". It is well known that people with "low and medium income" are more sociable and are more inclined to get together in groups, unions, clubs, parties, etc., while the sections of population behind the line of poverty – tramps, beggars – or, on the contrary, those having high and super-high income, trend to keep aloof: the first group in their slums, the second group in their palaces. Another example is people's rallying during the hard periods of their life: during the crises, wars and natural calamities. And on the contrary, they show striving to independence and self-sufficiency in the safe periods of prospering and well-being.

VI. Conclusion.

The method of description of system dynamics offered here has its advantages and disadvantages.

The advantages include the very possibility of mathematical description of the system evolution process as a process of realization of its "potential".

Many authors (for example, [9], [3] and etc.) mark the existence of universal principles of evolution of systems. We think that laws of evolution described by means of terms "potential" and "conditions" are just such universal principles.

In conclusion we would like to voice some considerations of philosophical character. One might think that the suggested approach will not have any practical value due to the abstract character of the terms used. One may also have some doubt as to the objective existence of the characteristic features underlying this approach. Does the substance that we call the "potential" and the "conditions" really exist? Can we attach the objective meaning to the logical structure existing only in our imagination? We are disposed to treat these questions in the positive manner. The reason is as follows. Analyzing the history of development of different systems, we, when trying to give a proper account of what is taking place, use the abstract terms, like "system potential", "development crisis", "progress", "regress", etc. Our thinking in this situation proceeds **within some logical structure**, within a certain system of categories by which we perceive and learn the existing reality as something comprehensible and sensible. The very fact that we comprehend this way and not the other way, using the selected concepts contrary to some other ones, just means that a certain Reality lies behind these notions, that there exists some inter-relational structure that may be revealed only at the level of such abstract terms. And *this Reality is functional*, in the sense that all systems emerging spontaneously in the natural environment and in the society evolve according to the laws inherent in this Reality. We ourselves are the same evolving systems, and laws of this Reality are realized through us as well setting the *logical structure* of the way of thinking specific for us. It is for that reason that the description of what is taking place in terms of this logical structure is perceived by us as the explanation. (Understanding of thinking (and its logical structure) as a result of evolution process is contained in work [6]).

There are some levels of the Reality. The first level is what is accessible to our organs of sense. The second level is made accessible to us by using certain appliances. The third level is the plane of abstract structures, which we access by way of thinking. This third level is as real as the other two. The Reality of the third level exists and functions, though lying beyond the zone of our sensual perception.

Where is the weak point of our approach? This is the absence of methods of correlation of abstract terms with the data being registrated. Obviously, such methods must exist. We must find the procedur es reducing the information about the system to two abstract parameters. However, this is subject for another labor-intensive work which yet ahead.

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