System Dynamics Analysis of Stability during Non-equilibrium Stage in Physical Distribution

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Abstract
As it is known, instability of inventory occurs due to lag in the information transmission in a distribution system. Excess reaction to the change in demand at each distribution sector is considered to be one of the main causes of this. Demand forecast is generated among multiple linked sectors, and as a result causes the development of estrangement between the actual demand and ordering by each sector. The complex behaviors exhibited by such chain systems have been thoroughly studied to date. However, in practice, all instabilities have not been eliminated. They have merely been tolerated within certain limits. This situation presumably suggests the existence of certain tolerable range in instabilities. This paper proposes to identify experimentally the assessment index measuring the quantification of the instability degree. Also it reports on the simulation that minimizes this instability region using the identified index through several example models.

Introduction
Many models are segmented into the basic loop of the divergence, convergence, and that behavior is analyzed with system dynamics. Identifying a basic loop, which dominates the system behavior, is considered to be essential in understanding the system. Discovery of the basic loop and understanding the system play important role in both quantitative and qualitative analysis.

However, predicting how the variable of the applicable problem changes is necessary for the general social science problem even under such conditions that the element, structure of the system are not well defined and only the value of the present and the past are known. In other words, there are numerous requirements to define the near range status from the data gathered with in limited time rather than over long period time. In particular, numerous modern social environments undergo rapid changes. Therefore, the period when one model can be applied onto a specific social phenomenon is limited. You may think that the social phenomenon exists in which the structure of the system changes rapidly. The standard which determines if a model is stable or unstable would be useful at present in this situation. Ford (1998) insists that the method made by the analysis of the basic loop is built on kind of clumsy factor "plausibility of the model" and "persuasive power of the analysis person". Then, he defined the behavior of each moment of the model into three kinds, convergent process, divergent process, and linear evolution process, based on the variable's second order derivative. Calculation of these parameters diachronically will make it possible to identify the dominant loop.

Many statistical techniques have already been proposed for a future prediction based on the past change. However, it is difficult to determine the mid or long term behavior of the system formed by more than one feedback loop since these techniques do not take the structural influence of the system into consideration. For example, when there are variables that are increasing with acceleration, it is difficult to determine whether this shift is due to simple one positive feedback or initial variation of S growth curve, unless efforts are made to determine
the structure. This shows the difficulty of determining the exponential behavior in the statistics treatment. Determining system structure from the data obtained by observing behavior provides necessary information to predict the behavior of the system.

Then, the tentative plan of two indexes is presented by this research. The pattern of the behavior is distinguished using these indexes as a preparation of examining how to distinguish structure from the data. One index compares the increase and decrease for a set time period. The second index compares the trend of present data with the one of the past data. These indexes show whether it is proceeding to the stable area or the unstable area. Firstly, the characteristic of these indexes for the typical structure which does not contain a vibration is shown and their validity is discussed.

**Index that Compares the Increase and Decrease for a Set Time Period**

The next index is based on the amount of increase of the data. This index is considered to be useful in determining whether present increase and decrease are exponential. It considers the following time series data (function value) which can be obtained from the model. 

\[ f(n+1), f(n), f(n-1), \ldots, f(2), f(1), f(0) \]

Then, the following time series of the increases is created from these.

\[ a(n+1) = |f(n+1) - f(n)|, a(n) = |f(n) - f(n-1)|, \ldots \quad (a \geq 0) \]

And totals for the following increases are taken.

\[ S(1) = a(n), S(2) = a(n) + a(n-1), S(i) = a(n) + a(n-1) + \ldots + a(n-i+1) \]

After that,

\[ a(n+1) > S(1) \]
\[ a(n+1) > S(2) \]
\[ a(n+1) < S(k) \]

and \( k \) is the index of \( f(n+1) \).

The calculation of this index was performed regarding logistic mapping. Logistic mapping possesses accelerative increase in the beginning. Then, the speed increase is lowered, and settles toward a certain objective value. Both the positive feedback loop and the negative feedback loop influence this behavior. Then, the influencing power shifts to the negative feedback loop from the positive feedback loop. The difference in \( a(t) \) and \( S(n) \) was obtained. The data are plotted in Figure 1.

Post \( t_4 \) is significant in Figure 1 because of the definition of the index \( S \). The influence of the positive feedback is significant, and it is understood that accelerative increase occurs in the early stage of the period between \( t_4 \) and \( t_{11} \). The negative degree is stronger for the indexes of the past three periods and of the two periods after that. This indicates that the influence of the negative feedback gets strong in comparison before.

![Figure 1: Past Increase vs. Present Increase (Logistic Mapping)](image-url)
Index that Compares the Trend of Present Data with the One of the Past Data

As for the time series data, the degree of the stability may be discussed by comparing a certain period or present behavior with behavior until now. In this case, a straight line to fit into the behavior of the majority data in the period until now is calculated. Then, the methodology which compares the coefficient describing this straight line with the coefficient describing the straight line of a certain period considered to be effective in discussing the degree of the stability.

The data y of the observation value is approximated in the linear line \( y=bt+c \) first. The method of least-squares is used for the data in approximation of linear type. The degree of change in the linear approximate equation slope b is used as an index. This index shows the degree of the stability.

b is calculated by the next formula.

\[
  b = \frac{(n \cdot t_i y_i - (\sum t_i)(\sum y_i))/(n \cdot t_i^2 - (\sum t_i)^2)}{n}: \text{the number of periods}
\]

The result of using this index for each of the following cases is shown; a positive feedback growth, an S curve growth, a negative feedback growth and a no feedback growth. \( b_3 \) is the slope of the approximation of linear type equation calculated from the value of the past three periods. \( b_5 \) is the slope of the approximation of linear type equation calculated from the value of the past five periods. It becomes a plus with the positive feedback loop when \( b_3-b_5 \) is acquired as index from these two (Figure 2).

Considering the slope of the approximation of linear type equation becoming negative, the equation of index is as follows.

\[
  \text{IF } (b_3\geq0) \text{ AND } (b_5\geq0) \text{ THEN } b_3-b_5 \text{ ELSE } \text{IF } (b_3<0) \text{ AND } (b_5<0) \text{ THEN } \text{ABS}(b_3) - \text{ABS}(b_5) \text{ ELSE } \text{ABS}(b_3) + \text{ABS}(b_5)
\]

This value becomes positive in the divergence part of the S curve growth, and becomes
negative in the part of the convergence (Figure 3). It becomes minus with the negative feedback loop (Figure 4). Then, it almost becomes a zero with the mere increase curve without a feedback loop (Figure 5).

The length of three and five periods is chosen suitably so that a period may be different. If five periods is taken longer, more past behavior is reflected on the index.

Figure 4: Negative Feedback Growth

Figure 5: Growth without Feedback

This index is thought to be effective when the stability of the movement of a certain period or the present is compared with past long-term behavior. As for Figure 6 and Figure 7, five periods is not used for b5 of index1, but it is extended in all the past periods. Then, the difference from b3 is calculated.

Figure 6: Unstable Area which does not Contain Convergence (t20•t25)
There are two behaviors of sample1 and sample2 doing a seemingly irregular movement. However, if sample2 is compared with sample1 about the region of \( t_{25} \) from \( t_{20} \), it can be presumed a stable area with the function which tries still to be settled with sample2 toward the behavior until now.

We continue to apply these indexes to more cases. These results as well as the possibility of other indexes will be examined further and reported.

References


