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A Step-by-step System Dynamics Modeling of Sustainability

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Abstract

The purpose of this paper is twofold. (1) A step-by-step procedure of system dynamics (SD) modeling is developed from a viewpoint of a mathematical system of difference equations. Through this procedure, essential concepts for building a SD model are developed such as the difference between a moment and a period of time, a unit check, a computational procedure for feedback loops, an expansion of boundaries, and a limit to an analytical mathematical model. (2) To exemplify the above procedure, a macroeconomic growth model is employed. Then a meaning of sustainability is clarified by expanding a model step by step from a simple macroeconomic growth model to a complicated ecological model. To be specific, sustainability is represented in terms of physical, social and ecological reproducibilities by a system of difference equations. As an implementation of the analysis, it is shown that a sustainable economic development is unsustainable in the long run with non-renewable resources being taken into the model.

1 A Macroeconomic Growth Model

An step-by-step construction of a system dynamics (hereafter called SD) model in this paper starts with a simple macroeconomic growth model which can

[*]This paper is presented at the 19th International Conference of the System Dynamics Society, Atlanta, Georgia, USA, July 23 – 27, 2001. The paper is mostly written while I'm visiting the Hawaii Research Center for Futures Studies, University of Hawaii at Manoa, in March, 2001. I truly thank Dr. James Dator, its Director, and his colleagues who kindly hosted my visit and provided a wonderful working environment for multidisciplinary studies. Hawaii turned out to be a good place for me to deeply consider sustainability. Since it became the 50th US state in 1959, only less than a half century has passed. Yet, its economy with 1.2 million islanders needs more than five million tourists and their food annually for its survival by dumping as much garbage. Can it be sustainable for this 21st century? Why did the Easter Island in the southern Pacific Ocean suddenly collapse: overpopulation, lack of food, water and natural resources, wars, epidemics ?

Table 1: Unknown Variables and Constants (1)

Category	Symbol	Notation	Unit
Unknown Variables	K_{t+1}	Capital Stock	machine
	Y_t	Output (or Income)	food/year
	C_t	Consumption	food/year
	S_t	Saving	food/year
	I_t	Investment	machine/year
Constants	v	Capital-Output Ratio (= 4)	machine/(food/year)
	c	Marginal Propensity to Consume (= 0.8)	dimensionless
Initial value	K_t	Initial Capital Stock (=400)	machine

be found in many macroeconomic textbooks. It consists of the following five equations.

$$K_{t+1} = K_t + I_t \quad \text{(Capital Accumulation)} \tag{1}$$

$$Y_t = \frac{1}{v} K_t \quad \text{(Production Function)} \tag{2}$$

$$C_t = cY_t \quad \text{(Consumption Function)} \tag{3}$$

$$S_t = Y_t - C_t \quad \text{(Saving Function)} \tag{4}$$

$$I_t = S_t \quad \text{(Equilibrium Condition)} \tag{5}$$

Equation (1) represents a capital accumulation process in which capital stock is increased by the amount of investment. Output is assumed to be produced only by capital stock in a macroeconomic production function (2). The amount of consumption is assumed to be a portion of output - a well-known macroeconomic consumption function (3). Saving is defined as the amount of output less consumption in (4). At the equilibrium investment has to be equal to saving as shown in (5), otherwise output would not be sold out completely or in a state of shortage.

These five equations become simple enough to describe a macroeconomic growth process. And most of the symbols used in the above equations should be familiar for economics and business students. Precise meaning of these variables, however, are usually left unexplained in the textbooks. SD modeling, on the other hand, requires precise specification of these variables, as defined in Table 1, without which it is impossible to construct a model. It is thus worth considering these specifications in detail.

Time

As emphasized in [8], it is fundamental in SD modeling to make a distinction between two different concepts of time. One concept is to represent time as a moment of time or a point in time, denoted here by τ; that is, time is depicted as a real number such that τ= 1, 2, 3, ... It is used to define the amount of stock at a specific moment in time. The other concept is to represent time as a period of time or an interval of time, denoted here by t, such that t = 1st, 2nd, 3rd, . . . , or more loosely t = 1, 2, 3, ... It is used to denote the amount of flow during a specific period of time. Units of the period could be a second, a minute, an hour, a week, a month, a quarter, a year, a decade, a century, a millennium, etc., depending on the nature of the dynamics in question. In a macroeconomic analysis, a year is usually taken as a unit period of time.

With these distinctions in mind, the equation of capital accumulation (1), consisting of a stock of capital and a flow of investment, has to be precisely described as

$$K_{\tau+1} = K_\tau + I_t \qquad \tau \text{ and } t = 2001, 2002, 2003, \cdots \tag{6}$$

A confusion, however, might arise from these dual notations of time, τ and t, no matter how precise they are. It would be better if we could describe stock-flow relation uniformly in terms of either one of these two concepts of time. Which one should, then, be adopted? A point in time τ could be interpreted as a limit point of the interval of time t. Hence, t can portray both concepts adequately, and usually be chosen.

When t is used to represent a unit interval between τ and $\tau + 1$, the amount of stock K_t thus defined at the t-th interval could be interpreted as an amount at the beginning point τ of a period t or the ending point $\tau + 1$ of the period t; that is,

$$K_t = K_\tau \qquad \text{Beginning amount of stock} \tag{7}$$

or

$$K_t = K_{\tau+1} \qquad \text{Ending amount of stock} \tag{8}$$

When the beginning amount of the stock equation (7) is applied, a stock-flow equation of capital accumulation (6) is rewritten as

$$K_{t+1} = K_t + I_t \qquad t = 2001, 2002, 2003, \cdots \tag{9}$$

In this formula, capital stock K_{t+1} is evaluated at the beginning of the period $t + 1$; that is, a flow of investment I_t is to be added to the present stock value of K_t for the evaluation of the capital stock at the next period.

When the ending amount of the capital stock equation (8) is applied, the stock-flow equation (6) is rewritten as

$$K_t = K_{t-1} + I_t \qquad t = 2001, 2002, 2003, \cdots \tag{10}$$

Two different concepts of time - a point in time and a period of time - are in this way successfully unified. It is very important for the beginners of SD modeling to understand that time in system dynamics usually implies a period of time which has a unit interval. Periods need not be discrete and could be continuous. In this paper, the beginning amount of capital accumulation (9) is employed as many macroeconomic textbooks do[1].

Unit

In SD modeling, units of all variables, whether unknowns or constants, have to be explicitly declared. In equation (5) of equilibrium condition, investment is defined as an amount of machine per year, while saving is measured by an amount of food per year. Therefore, in order to make the equation (5) congruous, a unit conversion factor ξ of a unitary value has to be multiplied such that

$$I_t = S_t * \xi, \tag{13}$$

in which ξ converts a food unit of saving to a machine unit of capital investment; that is to say, it has a unit of machine/food dimension. This tedious procedure of unit conversion could be circumvented by replacing machine and food units with a dollar unit as many macroeconomic textbooks implicitly presume so.

Model Consistency

A model consistency has to be examined as a next step in SD modeling, following time and a unit check. A model is said to be at least consistent if it has the same number of equations and unknown variables. This is a minimum requirement for any model to be consistent. The above macroeconomic growth model consists of five equations with five unknowns and two constants. Thus, it becomes consistent.

Let us now consider how these equations are computationally solved. Starting with the initial condition of the capital stock K_t, numerical values are assigned from the right-hand variable to the left-hand variables. We can easily trace these value assignments as follows:

$$K_t \to Y_t \to C_t \to S_t \to I_t \to K_{t+1} \tag{14}$$

This is how a computer solves equations of dynamic systems. .

[1]To show the difference between stock and flow explicitly, it would be informative to decompose the capital accumulation equation (1) as follows:

$$K_{t+1} \equiv K_t + \Delta K_t \qquad \text{(Idenatity of Capital Stock Accumulation)} \tag{11}$$

$$\Delta K_t = I_t \qquad \text{(Investment as a Flow of Capital)} \tag{12}$$

Feedback Loop

In our macroeconomic growth model, there are two types of equations. One type is the equation of *stock-flow relation* which specifies a dynamic movement. Capital accumulation equation (1) is of this type. The other type is the equation of *causal relation* in which a left-hand variable is caused by right-hand variables (and constants). The remaining four equations in the macroeconomic growth model are of this type.

These two types are clearly distinguished in SD modeling. A stock-flow relation is illustrated by a box that is connected by a double-lined arrow with a flow-regulating faucet, while a causal relation is drawn by a single-lined arrow. Then, we can easily trace a loop of arrows starting from a box and coming back to the same box. Such a loop is called *a feedback loop* in system dynamics. It is called *positive feedback* if an increase in stock results in an increase in a coming back stock, while *negative feedback* if an decreased amount of coming back stock results in.

A feedback loop corresponds with a computational trace of the equation (14). By drawing a SD model, we could easily find two such feedback loops that start from a capital stock box. A feedback loop has to include at least one stock-flow equation. Simultaneous equation system, on the other hand, has only equations of causal relation and, accordingly, cannot have feedback loops. Without a feedback loop, system cannot be dynamic.

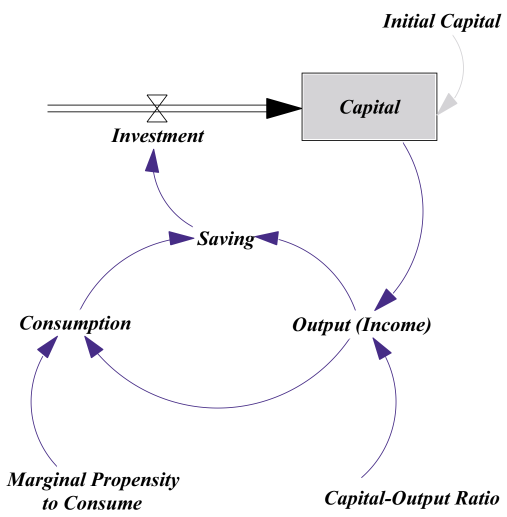

Figure 1: A Simple Macroeconomic Growth Model

A Steady State Equilibrium

Since SD modeling is by its nature dynamic, it is very important to find out a steady state equilibrium as a structural consistency of the model. Steady state implies that all stocks stop changing, which in turn means that values of flows (to be precise, net flows) become zero. In other words, it is a state of no growth. In our mode a steady state equilibrium of capital accumulation is attained for $K_{t+1} = K_t$. To attain the steady state analytically, equations of the model are first reduced to be a single equation of capital accumulation:

$$K_{t+1} = \left(1 + \frac{1-c}{v}\right) K_t. \tag{15}$$

Then, a steady state is easily shown to exit for $c = 1$; that is, output is all consumed and no saving is made available for investment. In our numerical example, a steady state equilibrium is attained at the values of $K^* = 400, Y^* = C^* = 100$, and $S^* = I^* = 0$.

Simulation for an Economic Growth

Let us try to drive the economy out of this steady state equilibrium. A growth path can be easily found by setting "a marginal propensity to consume" to be less than unitary; say, $c = 0.8$. Then 20% of output (or income) is saved for investment, which in turn increases the capital stock by the amount of 20, which then contributes to the increase in output by 5 next period, driving the economy toward an indefinite growth. Table 2 shows how capital, output, consumption and investment grow at the rate of 5% for $c = 0.8$.

Table 2: Macroeconomic Growth Model

Year	Capital	Output	Consump-tion	Invest-ment
2001	400.00	100.00	80.00	20.00
2002	420.00	105.00	84.00	21.00
2003	441.00	110.25	88.20	22.04
2004	463.04	115.76	92.61	23.15
2005	468.20	121.55	97.24	24.31
2006	510.51	127.62	102.10	25.52
2007	536.03	134.00	107.20	26.80
2008	562.84	140.71	112.56	28.14
2009	590.98	147.74	118.19	29.54
2010	620.53	155.13	124.10	31.02

Let us consider another growth path in which maximum amount of saving is made first at the cost of consumption, then, by accumulating capital stock as fast as possible, a higher level of consumption is enjoyed later. This type of growth path can be built by making "a marginal propensity to consume" as a function of a normalized output level such that

$$c = c(Y_t/Y_{norm}) \tag{16}$$

where Y_{norm} is a normalized reference level of output with which a current level of output is compared. Usually an initial value of output is selected as a reference level: $Y_{norm} = Y_{initial} = 100$. This function is called a table function, or a graphic function, or a lookup function in SD modeling.

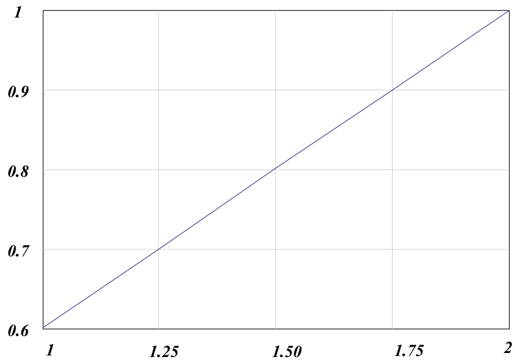

Figure 2: A Table Function of A Marginal Propensity to Consume

A simple example is the following linear function as illustrated in Figure 2:

$$c = 0.4\,\frac{Y_t}{Y_{initial}} + 0.2 \tag{17}$$

At the beginning "a marginal propensity to consume" is set to be a lowest (or a subsistence) level, say, $c(1) = 0.6$, to allow for a maximum growth rate, then it gradually becomes higher as income increases, enabling more consumption. When income level doubles, we have $c(2) = 1$ and no further saving and investment are made; that is, a maximum consumption level is enjoyed. Figure 3 illustrates a gradual increase in the value of "a marginal propensity to consume" and a gradual decrease in the growth rate.

Building up a table function is to connect the variable *Output* by a single-lined arrow to the constant *Marginal Propensity to Consume* in Figure 1. And a constant of "marginal propensity to consume" which has been residing outside the model now becomes an inside unknown variable whose value is to be determined by the behavior of the model itself. Better modeling is to reduce the number of constants and make a model self-determined by itself without relying on the outside values of constants. In this sense, a capability of introducing table functions is one of the most powerful features in SD modeling. In fact,

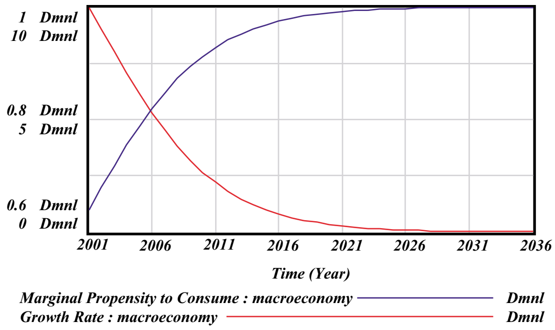

Figure 3: Growth Rate and Marginal Propensity to Consume

an introduction of nonlinear and/or numerical table functions can make many diversified dynamic behaviors possible for analytical simulations.

2 Physical Reproducibility

Sustainability

In the above macroeconomic growth model, depreciation of capital stock is not considered, or I_t is regarded as net investment. In reality, capital stocks depreciate, and for maintaining the current level of output, some portion of the income has to be saved to replace the depreciation. When a depreciation rate is high, a higher portion of income has to be saved at the cost of the consumption. Here arises a sustainability issue of the economy: how to maintain a level of income for sustainable development. In this sense a sustainability issue has been as old as human history.

Let us now consider what is meant by sustainability from an economic point of view. After the UN Conference on Environment and Development (UNCED), widely known as the *Earth Summit*, in Rio de Janeiro, Brazil, 1992, *sustainable development* becomes a fashionable word in our daily conversations. This might be an indication that our awareness on environmental crises such as global warming, acid rain, depletion of the ozone layer, tropical deforestation, desertification, and endangered species has deepened. How should, then, a state of sustainable development be defined? Some proposed definitions are the following:

> Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs. [4, p.43].

> The simplest definition is: A sustainable society is one that can persist over generations, one that is far-seeing enough, flexible enough,

> and wise enough not to undermine either its *physical* or its *social* systems of supports. (*Italic* emphases made by the author) [2, p.209].

These definitions are articulated so as to be understood even by children. However, from an economist's point of view, these definitions lack an interrelated view of production, consumption, society and environment.

A sustainability is comprehensively defined when all activities in economy, society and nature are interpreted as reproduction processes; that is, in terms of physical, social and ecological reproducibility [7]. A merit to this approach is that an economic structure such as in the general equilibrium framework [6] can be applied, since the most basic activity in any society is a reproduction process in which inputs are repeatedly transformed into outputs for consumption and investment each year. The same approach is followed in this paper. In this way the interrelationship between economic activities and environment is integrated wholistically.

Capital Depreciation

Let us introduce depreciation in the macroeconomic growth model. The equation of capital accumulation (1) is expanded as follows:

$$K_{t+1} = K_t + I_t - D_t \qquad \text{(Capital Accumulation)} \tag{18}$$

$$D_t = \delta K_t \qquad \text{(Capital Depreciation)} \tag{19}$$

As Figure 4 shows, this can be easily done in SD modeling by adding an outflow arrow of depreciation from the capital stock. I_t in equation (18) is now

Table 3: Unknown Variable and Constant Added (2)

New Variable	D_t	Depreciation	machine/year
New Constant	δ	Depreciation Rate (=0.02)	1/year

reinterpreted as gross investment.

Physical reproducibility implies that gross investment is greater than or equal to the depreciation.

$$I_t - D_t \geq 0 \qquad \text{[Physical Reproducibility]} \tag{20}$$

The macroeconomic growth model with depreciation, which is here called physical reproducibility model, now consists of 6 equations with 6 unknown variables: $K_{t+1}, Y_t, C_t, S_t, I_t, D_t$ and three constants: v, c, δ .

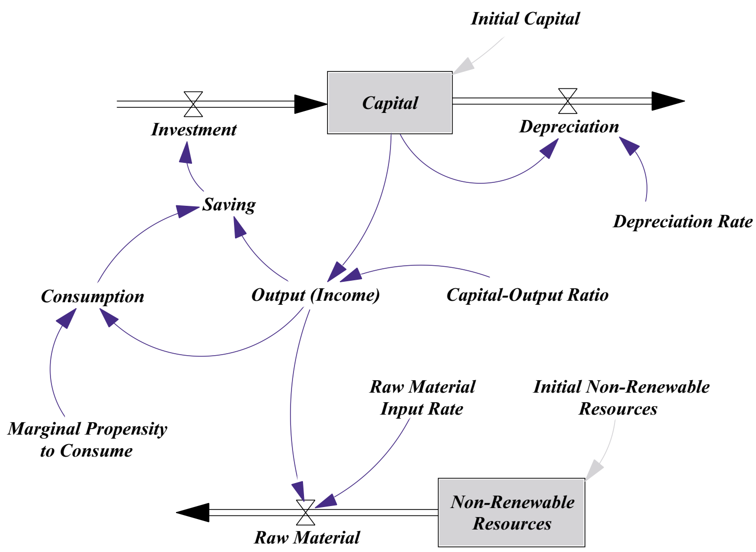

Figure 4: Physical Reproducibility Model

A Steady State Equilibrium

A steady state equilibrium is attained at $K_{t+1} = K_t$ or $I_t = D_t$, as easily shown from the equation (18). To obtain the steady state analytically, all equations in the model have to be reduced to a single capital accumulation equation:

$$K_{t+1} = \left(1 + \frac{1-c}{v} - \delta\right) K_t. \tag{21}$$

A steady state condition is then easily obtained as follows (asterisks are added to the constants that meet this condition):

$$\frac{1-c^*}{v^*} = \delta^* \tag{22}$$

At the steady state, "a marginal propensity to consume" becomes less than unitary; $c^* = 1 - \delta^* v^* < 1$, which implies that a portion of output has to be saved to replace the capital depreciation. One possible combination of numerical values for the steady state is $(v^*, c^*, \delta^*) = (4, 0.8, 0.05)$.

Simulation for an Economic Growth

For the economy to grow out of the steady state; that is, $K_{t+1} > K_t$, at least one of the following three actions has to be taken.

(1) Increase productivity ($\frac{1}{v} > \frac{1}{v^*}$) or $v < v^*$.

(2) Decrease consumption (or increase saving and investment) $c < c^*$.

(3) Improve capital maintenance $\delta < \delta^*$.

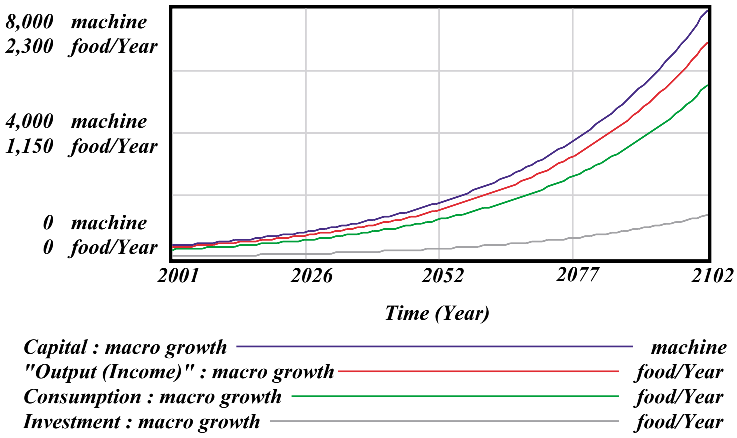

Figure 5: A Simulation for Economic Growth

As one such numerical example, let us take the case (3) and set a rate of depreciation at $\delta = 0.02$. In this case, an economic growth becomes 3%. As Figure 5 illustrates, during the 21st century capital stock keeps increasing from $K_{2001} = 400$ to $K_{2101} = 7,687$ and so does output from $Y_{2001} = 100$ to $Y_{2101} = 1,921$, more than 19 folds! Can such a growth be sustainable?

Non-Renewable Resource Availability

The physical reproducibility condition (20) presupposes an availability of non-renewable natural resources which is represented by the following equation:

$$R_{t+1} \equiv R_t - \Delta R_t \qquad \text{(Non-Renewable Resource Depletion)} \qquad (23)$$

$$\Delta R_t = \lambda Y_t \qquad \text{(Non-Renewable Raw Material Input)} \qquad (24)$$

For simplicity, let us here assume that non-renewable resources are represented by fossil fuels such as coal, gas, and oil whose units are uniformly measured by a ton. Then, λ is interpreted as an input amount of fossil fuels necessary for producing a unit of output.

Table 4: Unknown Variable and Constant Added (3)

New Variable	R_{t+1}	Non-Renewable Resource	ton
New Constant	λ	Raw Material Input Rate (=0.05)	ton/ food
Initial value	R_t	Initial Non-Renewable Resource (=1,00)	ton

Assuming that equations (23) and (24) are reduced to one equation, we have now 7 equations for 7 unknown variables and four constants. Hence the model is shown to be consistent.

Let us next consider the existence of a steady state equilibrium. There are two state variables K_{t+1} and R_{t+1} in the model. A steady state of capital accumulation is not affected by the introduction of non-renewable resources, while a steady state of non-renewable resources implies $R_{t+1} = R_t$, which in turn means $\Delta R_t = \lambda Y_t = 0$ or $Y_t = 0$. However, a steady state equilibrium of capital stock implies a positive amount of output; that is, $Y_t > 0$. A contradiction arises! Hence it is concluded that a macroeconomic growth model with non-renewable resources cannot have a steady state equilibrium by its nature. To make the model feasible, the existence of a steady state equilibrium of non-renewable resources has to be conceptually given up. Or, non-renewable natural resources have to be assumed to be available at any time in the economy so that the earth's limited source of non-renewable resources is not depleted; that is,

$$\sum_{t=2001}^{\infty} \Delta R_t < R_{2001} \qquad \text{[Non-Renewable Resource Availability]} \tag{25}$$

Simulation for Sustainability

Non-renewable resources are continuously deleted even at a steady state equilibrium of capital accumulation, contrary to a general belief that they are not in a non-growing economy.

At the steady state equilibrium set by the condition (22), as Figure 6 illustrates, the initial non-renewable resources $R_{2001} = 1,000$ constantly diminishes to one half a century later $R_{2101} = 500$. This can be easily examined by a simple calculation. Since the economy is at the steady state, the output level becomes constant at $Y_t = 100$. Hence, $\Delta R_t = \lambda Y_t = 0.05 \cdot 100 = 5$ and non-renewable resources are depleted by 5 tons every year. Over a century they are depleted by 500 tons. It is very important, therefore, to understand that *a steady state equilibrium is not sustainable in the long run.* In fact, a simple calculation shows that non-renewable resources will be totally exhausted over two centuries; that is, by the year 2201 we have $R_{2201} = 0$.

To show how fast non-renewable resources deplete under a growing economy, a depreciation rate is set to $\delta = 0.02$ and the economy starts growing at the

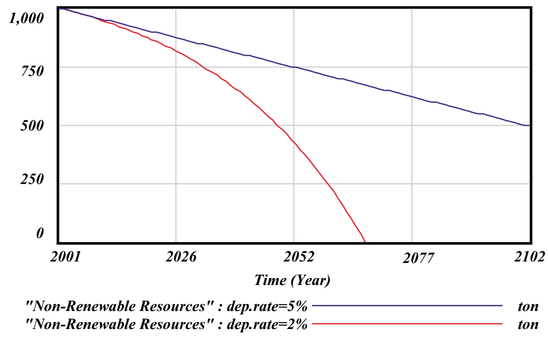

Figure 6: Depletion of Non-Renewable Resources

rate of 3%. In this case, non-renewable resources will be totally depleted in the year 2066; that is, at the beginning of the next year we have $R_{2067} = -5.813$, as Figure 6 illustrates.

How can we circumvent such a faster depletion of non-renewable resources and stay within a limit to resource availability and physical reproducibility? First, an efficient use of non-renewable natural resources has to be invented. For this, an introduction of long-term management of resources will be necessary. Second, substitutes for non-renewable resources have to be discovered or newly invented through technological breakthroughs. For this, research and development of new technology have to be oriented toward this direction. The issue of substitutes for non-renewable resources will be more fully analyzed in the next section.

Feedback Loop for Non-Renewable Resource Availability

What will happen if the development of substitutes are delayed or failed? To overcome a diminishing non-renewable resources, two self-regulating forces might appear in the economy. The first and more direct force is to curb down a raw material input rate λ. In a market economy, this might emerge as an increase in prices of non-renewable resources so that their use will be regulated. In SD modeling, this self-regulating force can be easily implemented by drawing an arrow from a stock of non-renewable resources to a constant of the raw material input rate λ and defining a table function as follows:

$$\lambda = \lambda \left(\frac{R_t}{R_{initial}} \right) \tag{26}$$

The second and more indirect force might appear as a reduction of productivity as non-renewable resources begin to be exhausted. In other words, a productivity which is defined as $\frac{1}{v}$ might begin to slide down. In SD modeling, this self-regulating force can be easily implemented by drawing an arrow from non-renewable resources to a capital-output ratio and defining a table function as follows:

$$v = v\left(\frac{R_t}{R_{initial}}\right) \tag{27}$$

In this paper, the second force of self-regulation is considered as an example of the effect of diminishing non-renewable resources on the economy. Let us assume that a productivity is not affected until non-renewable resources are depleted up to 40%. Then it begins to decrease as non-renewable resources continue to be depleted. Table 5 indicates one such numerical example of diminishing productivity (or an increasing capital-output ratio).

Table 5: A Table Function of Capital-Output Ratio

$R_t/R_{initial}$	0	0.1	0.2	0.3	0.4	0.5	0.6 - 1
v	20	16	12	8	6	5	4

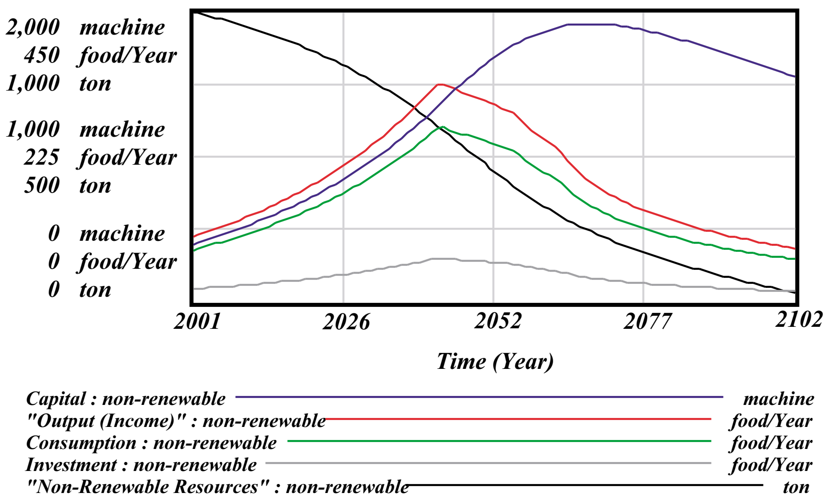

Figure 7: Physical Reproducibility with Non-Renewable Resources

Figure 7 illustrates the effect of such self-regulating forces. Output level attains its highest peak in the year 2043; $Y_{2043} = 337.53$, then begins to decrease. In the year 2093, the output level becomes less than its initial output level; $Y_{2093} = 98.15 < Y_{2001} = 100$. Apparently at this lower level of output the initial number of population would not be sustained. In other words, non-renewable resource availability and population growth become a serious trade-off, and either the preservation of non-renewable resources or population growth has to be sacrificed in the long run. To see this trade-off relation of sustainability more explicitly, the equation of population growth has to be brought into the model, which will be done in the next section.

3 Social Reproducibility

Population growth is embodied in the model as follows:

$$N_{t+1} \equiv N_t + \Delta N_t \qquad \text{(Population Growth)} \tag{28}$$

$$\Delta N_t = \alpha N_t - \beta N_t \qquad \text{(Net Birth = Birth - Death)} \tag{29}$$

Notations of new variables and constants are shown in Table 6.

Table 6: Unknown Variables and Constants Added (4)

New Variables	N_{t+1}	Population	person
	L_t	Workers (Labor Force)	person
New Constants	α	Birth Rate (= 0.03)	1/year
	β	Death Rate (= 0.01)	1/year
	θ	Participation Ratio to Labor Force(= 0.6)	dimensionless
	ℓ	Output-Labor Ratio (= 0.4)	(food/year)/person
	$\underline{c}$	Subsistence Consumption Level (= 0.16)	(food/year)/person
Initial value	N_t	Initial Population (=500)	person

For a survival of any society a minimum amount of consumption has to be at least produced each period to reproduce its population. This amount needs not be a subsistence amount, but has to be enough "to maintain the minimum standards of *wholesome and cultured living* (Article 25, The Constitution of Japan)." Let $\underline{c}$ be such a minimum amount of consumption per capita. Then, a total amount of consumption defined in the consumption function (3) has to be replaced with the following:

$$C_t = \underline{c} N_t \qquad \text{(Minimum Consumption)} \tag{30}$$

With the introduction of this minimum amount of consumption demanded irrespective of the output level, the amount of saving defined in the saving function (4) might become negative as population increases. To warrant a non-negative amount of saving, the saving function also has to be technically revised as follows:

$$S_t = \text{Max}\{Y_t - C_t, 0\} \qquad \text{(Non-Negative Saving)} \tag{31}$$

Social reproducibility is now defined as a reproduction process in which a minimum amount of consumption is always secured out of the net output[2]; that is,

$$Y_t - D_t - \underline{c}N_t \geq 0 \qquad \text{[Social Reproducibility]} \qquad (33)$$

Note that whenever this social reproducibility condition is met, physical reproducibility (20) also holds; that is,

$$I_t = S_t = Y_t - C_t = Y_t - \underline{c}N_t \geq D_t \qquad (34)$$

With the introduction of population, the number of workers or labor force is easily defined as a portion of the population:

$$L_t = \theta N_t \qquad \text{(Workers)} \qquad (35)$$

Production function (2) is then replaced with the following revised production function which allows an inclusion of labor force explicitly as a new factor of production[3].

$$Y_t = \text{Min}\{\frac{1}{v}K_t,\ \ell L_t\} \qquad \text{(Production Function)} \qquad (37)$$

A Steady State Equilibrium

Our macroeconomic growth model is now getting a little bit complex. From the tables of unknown variables and constants (1) through (4), 9 unknown variables and 9 constants are enumerated for 9 equations. Therefore, the model is shown to be consistent.

Let us now consider a steady state equilibrium. There are three variables of stocks such as capital, population and non-renewable resources; $K_{t+1}, N_{t+1}, R_{t+1}$. However, no steady state equilibrium is possible for non-renewable resources as already mentioned above. A steady state of population growth $N_{t+1} = N_t$ is

[2]To be precise, a unit of depreciation (machine/year) has to be converted to a unit of food (food/year) as follows:

$$Y_t - D_t/\xi \qquad (32)$$

as in the equation (13)

[3]Alternatively, a neoclassical production function such as a Cobb-Douglas production function can be used without any difficulty in SD modeling as follows:

$$Y_t = AK_t^a L_t^{1-a} \qquad (36)$$

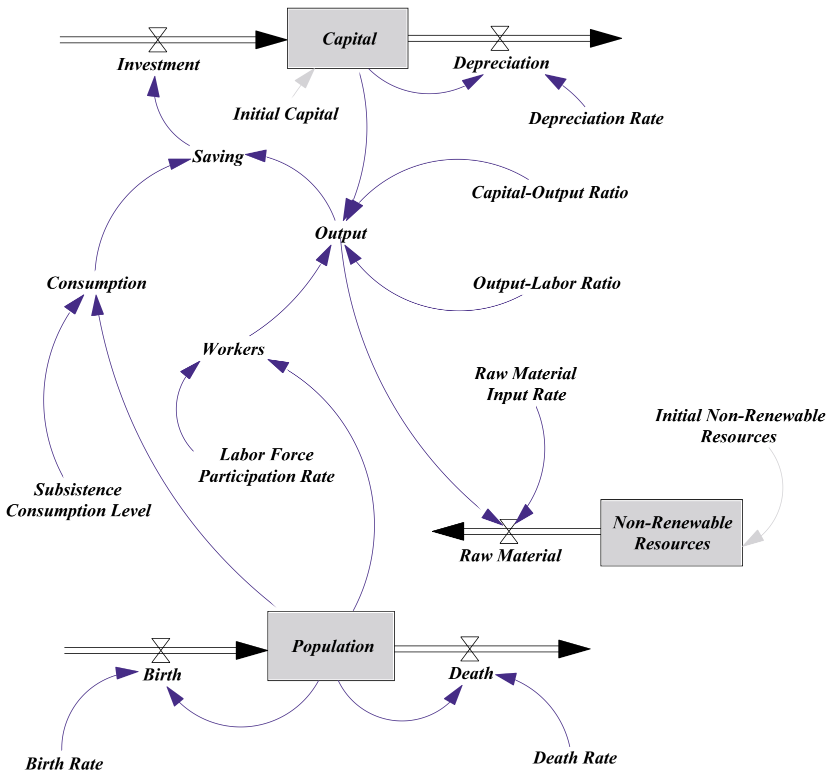

Figure 8: Social Reproducibility Model

attained when a birth rate is equal to a death rate; say, $\alpha^* = \beta^* = 0.01$. A steady state of capital stock can be attained as before for the values of constants; $(v^*, c^*, \delta^*) = (4, 0.8, 0.05)$. Due to the introduction of the new production function (37), two cases of steady state equilibria may emerge.

(1) A case in which output is constrained by capital stock: $Y_t = \frac{1}{v}K_t$. In this case, from a simple calculation we have

$$\frac{K_t}{N_t} = \frac{c}{\frac{1}{v} - \delta} = 0.8 \tag{38}$$

For $N_t = 500$, capital stock has to be $K_t = 400$ at the steady state. Hence, a steady state equilibrium is summarized as $(K^*, N^*, Y^*, C^*, S^*, I^*) = (400, 500, 100, 80, 20, 20)$, except that non-renewable resources keep depleting by the amount of 5 tons every year as analyzed under the previous section of physical reproducibility.

(2) A case in which output is constrained by workers: $Y_t = \ell L_t$. In this case, we have

$$\frac{K_t}{N_t} = \frac{\theta\ell - \underline{c}}{\delta} = 1.6 \tag{39}$$

For $N_t = 500$, capital stock this time has to be $K_t = 800$ at the steady state. Hence, a steady state equilibrium is summarized as $(K^*, N^*, Y^*, C^*, S^*, I^*) = (800, 500, 120, 80, 40, 40)$.

Neoclassical Golden Rule of Capital Accumulation

How are these two steady state equilibria related? To figure out these relations analytically, neo-classical economists invented a very neat concept of *per capita capital stock*, which is defined as $k_t = K_t/N_t$. Let $n(= \alpha - \beta)$ be a net growth rate of population. Then the above 8 equations, except for the non-renewable resources, are very compactly reduced to a single equation. To do so, the equation (18) is first rewritten as

$$\frac{K_{t+1}}{N_{t+1}}\frac{N_{t+1}}{N_t} = \frac{K_t}{N_t} + \left(\frac{Y_t}{N_t} - \frac{C_t}{N_t}\right) - \delta\frac{K_t}{N_t} \tag{40}$$

Then, a simple calculation results in the following capital growth equation.

$$k_{t+1} = k_t + \frac{1}{1+n}\left(Min\{\frac{1}{v}k_t,\ \theta\ell\} - \underline{c} - (n+\delta)k_t\right) \tag{41}$$

A steady state equilibrium is obtained at $k_{t+1} = k_t$, which in turn yields two equilibrium levels of per capita capital.

For a smaller level of equilibrium we have

$$\underline{k} = \frac{\underline{c}}{\frac{1}{v} - (n+\delta)} \tag{42}$$

This corresponds to the previous case in which output is constrained by capital stock (38). For a larger level of equilibrium we have

$$k^* = \frac{\theta\ell - \underline{c}}{n+\delta} \tag{43}$$

This corresponds to the previous case in which output is constrained by the number of workers (39). Note that even at a steady state of per capita equilibrium, population is allowed to grow at a net growth rate of $n > 0$.

It is easily shown that $\underline{k}$ is an unstable state of equilibrium, since $k_{t+1} < k_t$ for $k_t < \underline{k}$, and $k_{t+1} > k_t$ for $\underline{k} < k_t < k^*$. Thus, per capita capital k_t is, once displaced with the equilibrium, shown to decrease toward zero or converge to k^*. Meanwhile, k^* is a stable state of equilibrium, since $k_{t+1} < k_t$ for $k_t > k^*$.

$$k_t \longrightarrow \begin{cases} 0, & \text{if } k_t < \underline{k} \\ k^*, & \text{if } k_t > \underline{k} \end{cases} \tag{44}$$

Let us examine the stability of per capita capital numerically by allowing the economy to grow out of the initial steady state equilibrium. Depreciation and birth rates are now reset to $(\delta, \alpha) = (0.02, 0.03)$ so that both capital stock and population are allowed to start growing. It is then calculated that $\underline{k} = 0.7619$ and $k^* = 2$. Since the initial population is 500, an unstable equilibrium level of initial capital stock is obtained as $K_{2001} = \underline{k}N_{2001} = 380.95$. This means that if the initial capital stock is less than this amount, per capita capital stock tends to diminish toward zero, and the economy will get stuck eventually. Figure 9 numerically illustrates that when $K_{2001} = 380$ (and $k_{2001} = 0.76$) per capita capital decreases to $k_{2100} = 0.0296$, and eventually to zero.

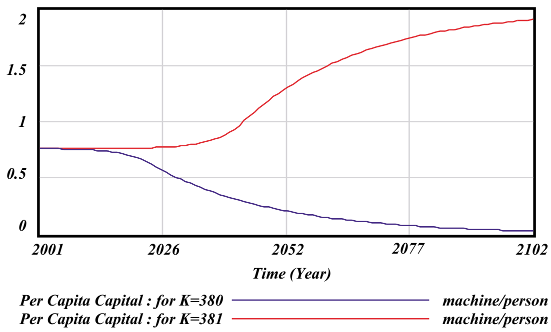

Figure 9: Golden Rule of Capital Accumulation

On the other hand, if the initial capital stock is greater than this amount, per capita capital tends to converge towards a so-called *golden-rule level of capital* in neoclassical growth theory [1]. Figure 9 also illustrates that when $K_{2001} = 381$(and $k_{2001} = 0.762$) per capita capital increases to $k_{2100} = 0.1898$, and eventually converges to a golden rule level of capital: $k^* = 2$.

It is interesting to know that only one unit difference of capital stock in our numerical example will result in a big difference in the growth paths. When the initial capital stock is $K_{2001} = 380$, the economy will be destined to be trapped forever to a stagnant state, while an additional unit of capital stock will drive the economy to its prosperity. The importance of an initial level of capital stock for an economic development is a well recognized feature in development economics. To circumvent the situation of this economic trap, the initial capital stock is set so far to $K_{2001} = 400$ in our numerical example.

From now on let us reset the initial value of capital stock, without losing generality, at its critical value of $K_{2001} = 381$. Figure 10 illustrates how capital stock, population, output, consumption and investment grow simultaneously. Population grows at 2%, so does a minimum amount of consumption regardless of the growth of capital stock and output level. Output is first constrained by the availability of capital stock, then from the year 2042, it is constrained by the availability of workers, which is in turn constrained by the population growth. Thus, the economy continues to grow at an increasing rate as the capital stock grows up to the year 2042 (from 2% to 5%), then it grows at a constant rate of population growth of 2% . This is why there are some bumps on the output and investment growth paths around 2042.

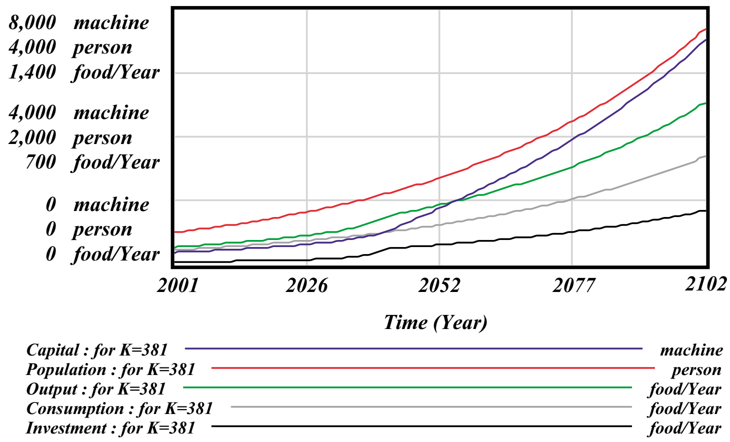

Figure 10: Golden Rule of Economic Growth

Even at this growth rate of population, output level is still maintained at a higher level than a minimum amount of consumption so that social reproducibility is constantly sustained. Eventually, per capital capital growth will converge to a steady state of k^*, showing a long-run stability of capital accumulation. This is what is meant by a neoclassical economic growth of golden rule: a very elegant and optimistic theory of economic growth!

Can such a growth be sustainable in the long run, indeed? The answer would be yes as long as non-renewable resources are disregarded and left out of the model. Remembering, however, that neoclassical concept of a steady state allows a constant growth rate of 2% , and the economy still keeps growing, depleting non-renewable resources, the answer would be absolutely no. In fact, non-renewable resources will be totally depleted in the year 2077 in our numerical example and become negative for the next year; that is, $R_{2078} = -13.31$. Even so, neoclassical growth theory keeps silent about this point, giving the impression that our macroeconomy can continue to grow and be stable in the long run.

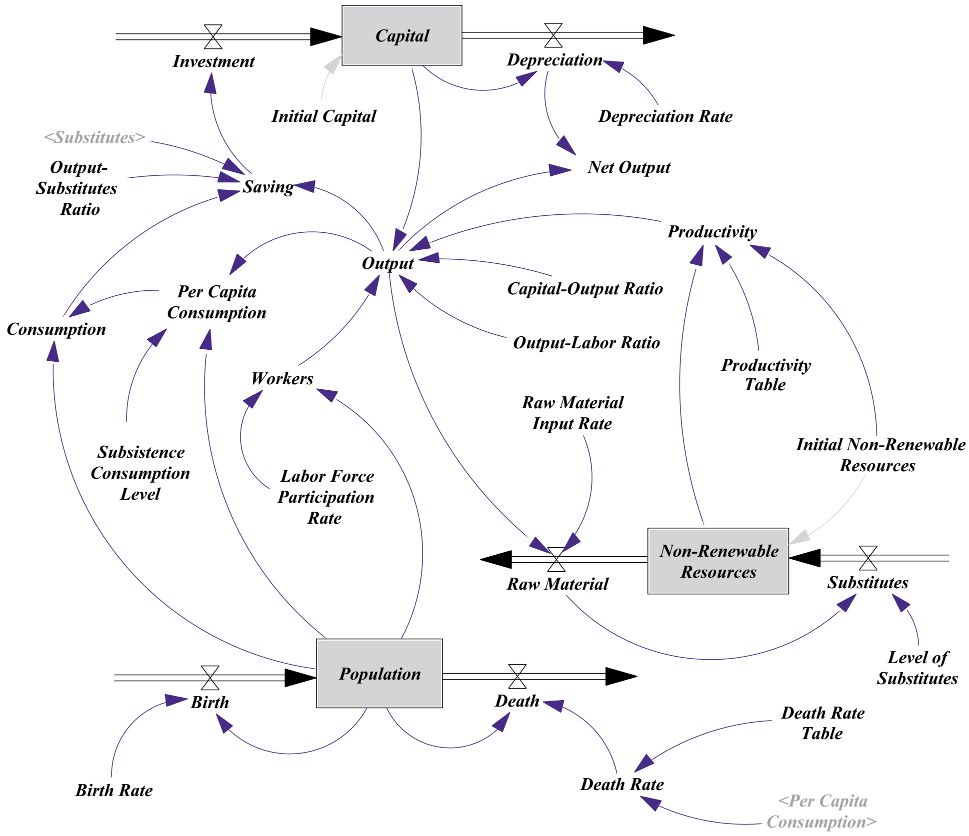

Figure 11: Social Reproducibility Feedback Model

Feedback Loop for Non-Renewable Resource Availability

Availability of non-renewable resources is now taken into consideration. To make non-renewable resources available for future generations, let us introduce a similar feedback mechanism as implemented in the previous section of physical reproducibility; that is, as the non-renewable resources continue to be depleted, productivity becomes worsened, and accordingly output is curbed, resulting in preserving non-renewable resources. Two constants in the production function (37) could influence productivity separately; that is, capital-output ratio v and output-labor ratio ℓ. Instead of affecting these separately, we introduce a table function which affects output level directly such that

$$Y_t = \text{Productivity}\left(\frac{R_t}{R_{initial}}\right)\text{Min}\{\frac{1}{v}K_t,\ \ell L_t\} \quad \text{(Production Feedback)} \tag{45}$$

A table function of the productivity is defined in Table 7. It is assumed that the productivity is not affected until non-renewable resources are depleted up to 40% , beyond which, then, it begins to decrease gradually. Figure 11 shows a revised feedback loop for non-renewable resources.

Table 7: A Table Function of Productivity

$R_t/R_{initial}$	0	0.1	0.2	0.3	0.4	0.5	0.6 - 1
Productivity	0	0.1	0.2	0.4	0.6	0.8	1

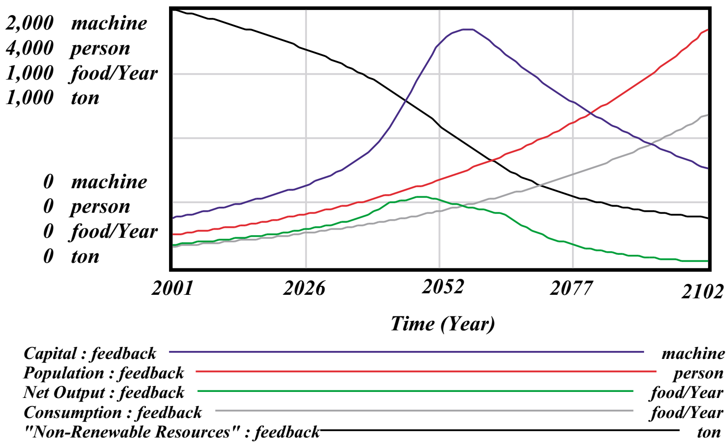

Figure 12: Golden Rule with Non-Renewable Resource Feedback

As expected by the introduction of a productivity feedback loop, Figure 12 illustrates how growth paths of capital stock and net output (output less depreciation) are curved as non-renewable resources continue to be depleted. In this way non-renewable resources are to be preserved.

Feedback Loop for Social Reproducibility

Figure 12 also illustrates that population increases exponentially at a net growth rate of 2% , so does a minimum amount of consumption for maintaining a per-capita wholesome and cultured living standard: $C_t = \underline{c}N_t$. Since net output is curved by a negative feedback loop of non-renewable resources, social reproducibility condition (33) will be eventually violated, and a portion of the population might be forced to be starved to death.

The violation of social reproducibility implies

$$Y_t - D_t - \underline{c}N_t < 0. \tag{46}$$

In our numerical example, this occurs in the year 2057 when $C_{2057} = 242.49$ and $Y_{2057} - D_{2057} = 234.33$, so that consumption exceeds net output by the amount of 8.16 as roughly illustrated in the Figure 12. The violation of social reproducibility implies that a smaller amount of net output has to be shared among people, forcing their level of living standards to be reduced. How far can such a per capita consumption be lowered? For maintaining physical reproducibility, it is desirable to keep its level at which per capita consumption is equal to per capita net output. It would be imaginable, however, that starving people would eat up everything available out of the output, including the reserved amount of capital stock for depreciation.

Reflecting the situation of food shortage, per capita consumption is recalculated as follows[4]:

$$\text{Per Capita Consumption} = \text{Min}\left(\underline{c},\ \frac{Y_t}{N_t}\right) \tag{48}$$

This formula enables per capita consumption to be lowered from the level of $\underline{c} = 0.16$. A decrease in per capita consumption may increase a death rate due to a food shortage. This could happen equally among weaker people and children, or among the countries whose economy is not wealthy enough to buy food, or among the countries that are politically weaker and neglected. A table function of death rate in Table 8 is created to reflect such situations. For

Table 8: A Table Function of Death Rate

Per Capita Consumption	$\underline{c} = 0.16$	0.14	0.12	0.1	0.08	0.06	0.04
Death Rate β	0.01	0.015	0.02	0.03	0.05	0.07	0.1

instance, whenever a per capita consumption is reduced by half from the original minimum amount, a death rate is assumed to jump to 5% from 1% . In this way, a negative feedback loop of social reproducibility is completed. Figure 11 shows a revised feedback version of social reproducibility model.

Figure 13 illustrates revised growth paths that reflect the feedback loop relation to the death rate. The amount of consumption exceeds net output during the year 2056, and accordingly capital stock begins to decay. The difference between consumption and net output is the amount of capital depreciation that is allowed to be consumed by hungry people.

[4] On the other hand, for maintaining the physical reproducibility, the equation of per capita consumption has to be changed to the following:

$$\text{Per Capita Consumption} = \text{Min}\left(\underline{c},\ \frac{Y_t - D_t}{N_t}\right) \tag{47}$$

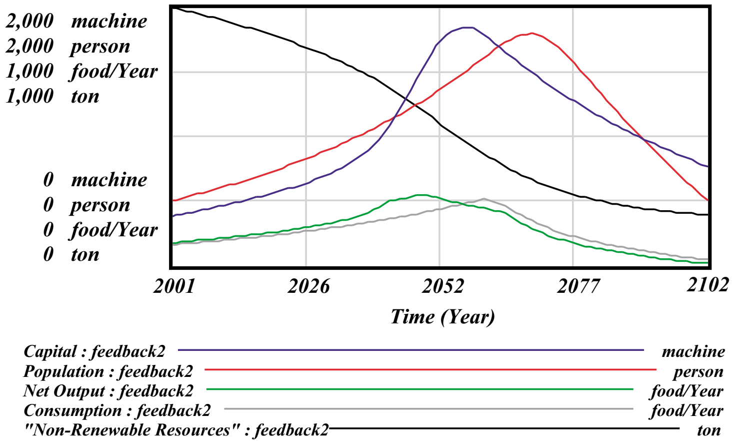

Figure 13: Growth Paths with Social Reproducibility Feedback

A century later, output level becomes only one quarter of its initial level; that is, $Y_{2101} = 25.71$ from $Y_{2001} = 95.25$. Population is almost pulled back to its original level of $N_{2001} = 500$; that is, it increases to its peak at $N_{2069} = 1,793$, then begins to decline to $N_{2101} = 541.51$. Per capita income level has been maintained at $\underline{c} = 0.16$ until the year 2059, then begins to decline to the level of 0.0474 in the year 2101 (a 70% decrease !), and the death rate jumps up to almost 10 %. In this way, all economic activities will be trapped. Is there a way to escape from this economic trap?

Substitutes for Non-Renewable Resources

The economic trap mentioned above is basically caused by a diminishing availability of non-renewable resources. To see the effect, let us modify the equation of non-renewable resource depletion (23) so that it allows an inflow of substitutes for non-renewable resources. Let SU_t be an inflow amount of non-renewable

Table 9: Unknown Variable and Constant Added (5)

New Variable	SU_t	Non-Renewable Substitutes	ton/year
New Constant	ν	Level of Substitutes ($0 \leq \nu < 1$)	dimensionless
	ρ	Output-Substitutes Ratio (=1)	food/ton

substitutes, measured by a unit of ton/year, that can be added to the stock of non-renewable resources, and ν be a level of the substitutes such that $0 \leq \nu < 1$.

Then the equation (23) is replaced with the following:

$$R_{t+1} = R_t + SU_t - \Delta R_t \qquad \text{(Non-Renewable Resource Depletion)} \tag{49}$$

$$SU_t = \nu \Delta R_t \qquad \text{(Substitutes for Non-Renewable Input)} \tag{50}$$

Or, combining these two, we have

$$R_{t+1} = R_t - (1-\nu)\Delta R_t \qquad \text{(Non-Renewable Resource Depletion)} \tag{51}$$

Where do the substitutes come from? For simplicity it is assumed that they are converted from the output by a factor of output-substitutes ratio. Saving function (4) then has to be revised as follows:

$$S_t = Y_t - C_t - \rho SU_t \tag{52}$$

Figure 11 illustrates the SD modeling implementation of the substitutes for non-renewable resources.

Several simulations are done, under such circumstances, to attain the growth paths of the golden rule of capital accumulation illustrated in Figure 10. It turned out that at least 400 unit machines of initial capital stock are needed to drive an economic growth initially. So the initial capital stock is reset again to $K_{2001} = 400$. Even so, if a level of substitutes is set high above 80%, the economy again turns out to be trapped. This is a little bit surprising result, because a higher rate of substitutes is supposed to preserve the non-renewable resources. A moment of thought clarifies the reason. A higher level of substitutes subtracts more portion of output, and capital accumulation begins to decline with less saving and investment. On the other hand, a lower level of substitutes depletes non-renewable resources faster, reducing productivity and output. Again, the economy is trapped.

If a level of substitutes is 80%, the economy can recover from the economic trap that is caused by a negative feedback loop of non-renewable resources as illustrated in Figure 13, and once again attain the growth paths of golden rule for the entire 21st century. Figure 14 illustrates such golden rule growth paths up around the turn of the 21st century.

However, this is nothing but postponing a problem of economic trap to the 22nd century, and there will be no way to escape from the economic trap in the long run. Figure 14 shows exactly the same structure as in the Growth Paths with Social Reproducibility Feedback in Figure 13, except that a time scale is elongated over two centuries in the present case. Substitutes of non-renewable resources cannot be an economic savior in the long run.

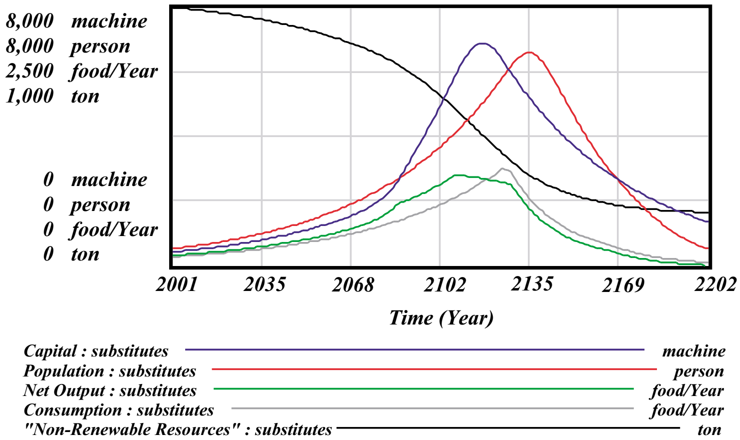

Figure 14: Social Reproducibility Economic Trap in the Long Run

4 Ecological Reproducibility

Production and consumption activities as well as capital accumulation formalized above produce as by-products consumer garbage GC_t, industrial wastes GY_t and capital depreciation dumping GK_t. These by-products are in turn dumped into the earth or scattered around atmosphere and accumulated as an artificial environmental stock called sink SK_{t+1}. Some portion of the sink will be naturally regenerated (or recycled) and made available as renewable resource stock that is called source SR_{t+1}. As a typical example, we can refer to photosynthesis processes in which tropical forests and trees grow by taking carbon dioxides (industrial wastes) as inputs and producing oxygen as by-product output.

These three dumping processes together with an extracting process of non-renewable resources now form an entire global environment Env, consisting of the earth's sink and source. Hence, the formation of the entire global environment might be appropriately considered as an ecological reproduction process which is symbolically illustrated as:

$$(\ominus \Delta R_t \oplus GC_t \oplus GY_t \oplus GK_t) \Longrightarrow Env(SK_{t+1} \rightarrow SR_{t+1}). \qquad (53)$$

To describe such an ecological reproduction process, we need to add the following seven equations.

$$
\begin{aligned}
SK_{t+1} &\equiv SK_t + \Delta SK_t && \text{(Accumulation of Sink)} && (54)\\
\Delta SK_t &= GC_t + GY_t + Gk_t - (\epsilon + \mu)SK_t && \text{(Net Change in Sink)} && (55)\\
GC_t &= \gamma_c C_t && \text{(Consumer Garbage)} && (56)\\
GY_t &= \gamma_y Y_t && \text{(Industrial Wastes)} && (57)\\
GK_t &= \gamma_k D_t && \text{(Depreciation Dumping)} && (58)\\
SR_{t+1} &\equiv SR_t + \Delta SR_t && \text{(Accumulation of Source)} && (59)\\
\Delta SR_t &= (\epsilon + \mu)SK_t - \lambda_1 Y_t && \text{(Net Change in Source)} && (60)
\end{aligned}
$$

Table 10: Unknown Variables and Constants Added (6)

New Variables	SK_{t+1}	Sink	source
	SR_{t+1}	Source	source
	GC_t	Consumer Garbage	source/year
	GY_t	Industrial Wastes	source/year
	GK_t	Capital Depreciation Dumping	source/year
New Constants	ϵ	Natural Rate of Regeneration (= 0.15)	1/year
	μ	Recycling Rate (= 0.05)	1/year
	λ_1	Renewable Raw Material Input Rate (= 0.6)	source/food
	γ_c	Garbage Rate(= 0.5)	source/food
	γ_y	Industrial Wastes Rate (= 0.1)	source/food
	γ_k	Depreciation Dumping Rate (= 0.5)	source/machine
Initial values	SK_t	Initial Sink (=300)	source
	SR_t	Initial Source (=3,000)	source

In order for an ecological reproduction process to continue, total amount of consumer garbage, industrial wastes and capital depreciation dumping have to be less than the earth's ecological capacity to absorb and dissolve the sink, and those newly regenerated source have to add enough amount to renewable source for continued production activities. Otherwise, the amount of sink begins to accumulate, and the accumulated sink will eventually cause the environment to collapse, or renewable source will be completely depleted. Therefore, for a sustainable ecological reproducibility, the following two conditions have to be met.

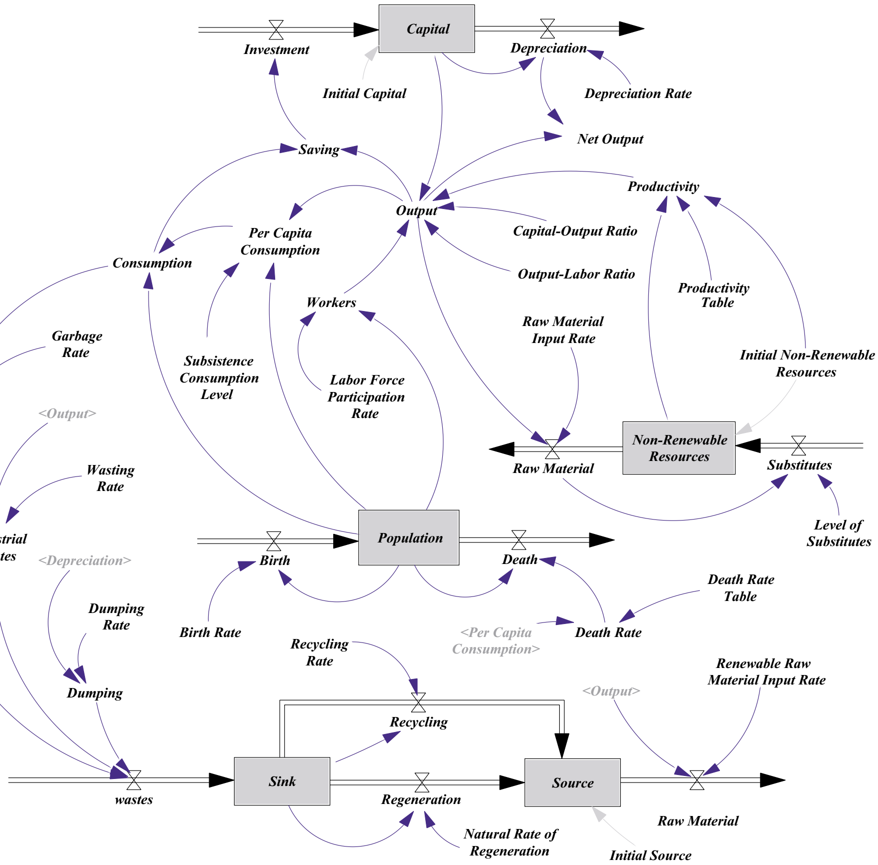

Figure 15: Ecological Reproducibility Model

$$\sum_{t=2001}^{\infty} (GC_t + GY_t + GK_t) \leq \epsilon \sum_{t=2001}^{\infty} SK_{t+1} \quad \text{[Ecological Reproducibility]} \tag{61}$$

$$SR_{t+1} > 0,\ t = 2001,\ \cdots \quad \text{[Renewable Resource Availability]} \tag{62}$$

Fortunately, the ecological reproducibility of recycling sink into source and restoring the original ecological shape has been built in the earth as a self-regulatory mechanism of Gaia [3]. Consumer garbage, industrial wastes and capital depreciation dumping have been taken care of and disintegrated by a natural reproduction process, and the environment so far seems to have continued to restore itself to a certain degree. Therefore, a sustainable development might be possible for the time being so long as the accumulated sink which the ecological reproduction process fails to disintegrate does not reach the environmental capacity of regeneration.

As production and consumption activities expand exponentially, however, such environmental sink also continues to accumulate exponentially. And naturally built-in ecological reproducibility of Gaia eventually begins to fail to regenerate the sink so that a portion of the sink will be left unprocessed Eventually, an environmental catastrophe occurs, and the earth might become uninhabitable for many living species, including human beings. In fact, many environment scientists warn us that such a catastrophe has already begun. For instance, see [5].

Accordingly, to be able to stay within a limit to ecological reproducibility, first of all, the total amount of environmental sink has to be directly regulated within an environmental regenerating capacity. Second, new development of recycling-oriented products has to be encouraged so that the amount of environmental sink is reduced at every cycle of reproduction and consumption process. Third, hazardous and toxic wastes which are not naturally disposed of have to be chemically processed and recycled safely at all costs. Then, the equation of ecological reproducibility (61) is expanded as follows:

$$\sum_{t=2001}^{\infty} (GC_t + GY_t + GK_t) \leq (\epsilon + \mu) \sum_{t=2001}^{\infty} SK_{t+1} \quad \text{[Recycling of Sink]} \tag{63}$$

A Steady State Equilibrium

A steady state equilibrium of of the ecological reproducibility is attained at $SK_{t+1} = SK_t$ and $SR_{t+1} = SR_t$; that is, $\Delta SK_t = \Delta SR_t = 0$. From the above equations of ecological reproducibility this implies

$$GC_t + GY_t + Gk_t = (\epsilon + \mu)SK_t = \lambda_1 Y_t \tag{64}$$

A steady state of capital accumulation is already obtained under the section of physical reproducibility. Using the same numerical values of that steady state, and constant values assigned in Table 10, we have

$$GC_t + GY_t + Gk_t = 0.5 \cdot 80 + 0.1 \cdot 100 + 0.5 \cdot 20 = 60. \tag{65}$$

$$(\epsilon + \mu)SK_t = (0.15 + 0.05)300 = 60. \tag{66}$$

$$\lambda_1 Y_t = 0.6 \cdot 100 = 60. \tag{67}$$

A steady state of population growth is attained when rates of birth and death are equal as shown under the section of social reproducibility. Hence, a steady sate of ecological reproducibility is shown to exist and our model of the ecological reproducibility becomes consistent. However, this is no longer true if non-renewable resources are considered explicitly. Figure 16 illustrates that an

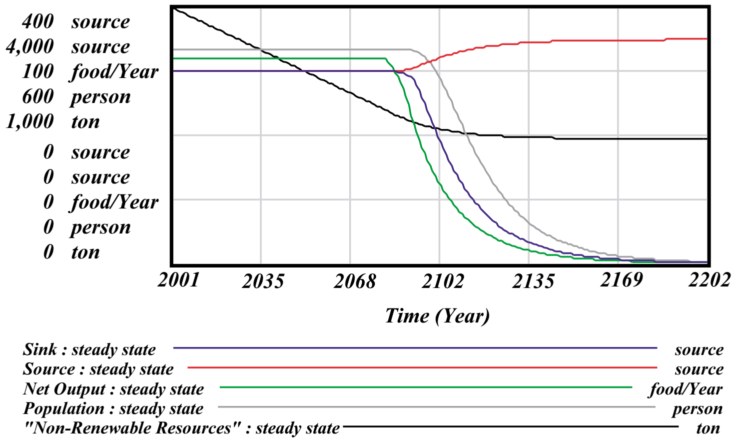

Figure 16: A Steady State of Ecological Reproducibility

ecological steady state equilibrium is sustained almost throughout the 21st century until net output starts decreasing in the year 2082. This decrease in the net output is caused by a diminishing productivity, which is in turn caused by the depletion of non-renewable resources. Accordingly, per capita consumption decreases and a death rate increases, resulting in a decline of population growth that begins to start in the year 2091, a decade later. Hence, an ecological steady state equilibrium becomes impossible in the long run if non-renewable resources are taken into consideration.

Simulations for Sustainable Growth

When a depreciation rate and a birth rate are reset at the original values; that is $\delta = 0.05$ and $\alpha = 0.03$, respectively, the economy begins to grow. However, this growth paths are eventually curbed by the depleting non-renewable resources as illustrated in Figure 17.

To avoid such restrictions of the growth paths, a level of substitutes might be set to be 80% as in the previous section. Then the net output and population once again continue to grow for the entire 21st century. However, this sustained growth paths begin to cause a problem of ecological unsustainability. The amount of sink continues to accumulate and source is completely depleted in the year 2077 as illustrated in Figure 18.

Eventually some negative feedback loops might emerge to prevent such environmental catastrophes. For instance, an over-accumulated amount of the sink such as chemical wastes will surely affect human health and a birth rate will be reduced as a result: a feedback loop from the sink to the birth rate. Meanwhile, as renewable source continue to be depleted, output will be curbed as in the case

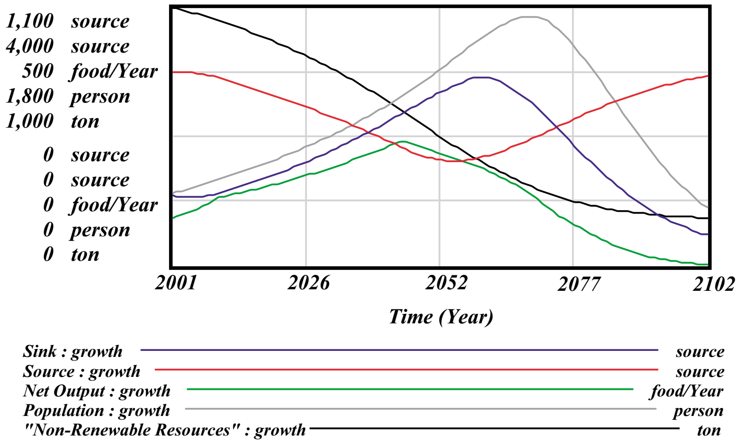

Figure 17: Growth Paths of Ecological Reproducibility

of the depletion of non-renewable resources: a feedback loop from the source to output.

Such loops can be built by introducing appropriate table functions. In this way an ecological reproducibility might be restored by avoiding the problems of the over-accumulation of the sink and the depletion of the source. Due to a shortage of the space assigned to this paper, we only show one such sustainable growth paths in Figure 19 without specifying here numerical values of table functions for the negative feedback loops. With the introduction of 80% level of substitutes for non-renewable resources together with ecological feedback loops, there exists indeed ecological sustainable paths over the next two centuries. It is worth warning, however, that the economic trap illustrated in Figure 14 will eventually emerge in the 23rd century so long as non-renewable resources continue to be depleted!

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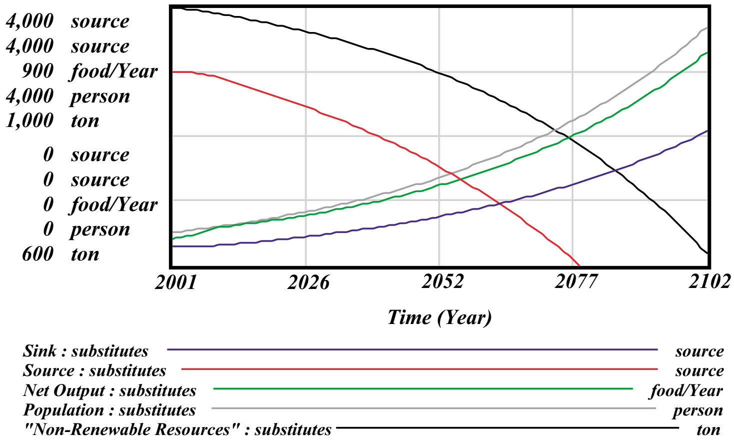

Figure 18: Growth Paths with Non-Renewable Substitutes

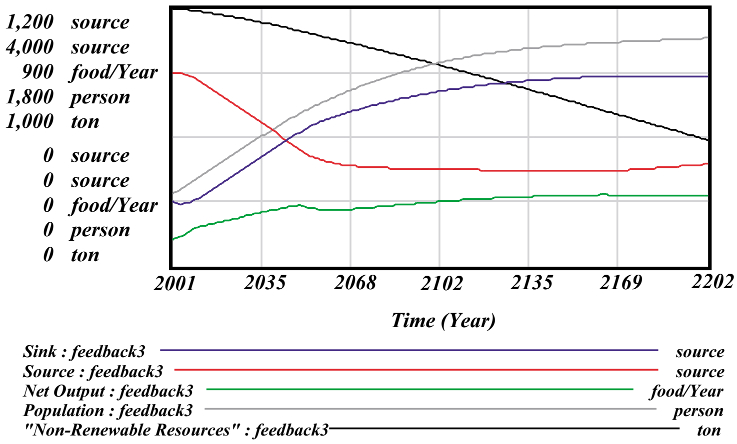

Figure 19: Growth Paths with Ecological Feedback